

ASSESSING APPROXIMATE ARITHMETIC DESIGNS IN THE PRESENCE OF PROCESS
VARIATIONS AND VOLTAGE SCALING

by

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B.Tech. B S Abdur Rahman University, 2013

A thesis submitted in partial fulfilment of the requirements
for the degree of Master of Science
in the Department of Electrical Engineering and Computer Science
in the College of Engineering and Computer Science
at the University of Central Florida
Orlando, Florida

Spring Term
2015

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ABSTRACT

As environmental concerns and portability of electronic devices move to the forefront of priorities, innovative approaches which reduce processor energy consumption are sought. Approximate arithmetic units are one of the avenues whereby significant energy savings can be achieved. Approximation of fundamental arithmetic units is achieved by judiciously reducing the number of transistors in the circuit. A satisfactory tradeoff of energy vs. accuracy of the circuit can be determined by trial-and-error methods of each functional approximation. Although the accuracy of the output is compromised, it is only decreased to an acceptable extent that can still fulfill processing requirements.

A number of scenarios are evaluated with approximate arithmetic units to thoroughly cross-check them with their accurate counterparts. Some of the attributes evaluated are energy consumption, delay and process variation. Additionally, novel methods to create such approximate units are developed. One such method developed uses a Genetic Algorithm (GA), which mimics the biologically-inspired evolutionary techniques to obtain an optimal solution. A GA employs genetic operators such as crossover and mutation to mix and match several different types of approximate adders to find the best possible combination of such units for a given input set. As the GA usually consumes a significant amount of time as the size of the input set increases, we tackled this problem by using various methods to parallelize the fitness computation process of the GA, which is the most compute intensive task. The parallelization improved the computation time from 2,250 seconds to 1,370 seconds for up to 8 threads, using both OpenMP and Intel TBB. Apart from using the GA with seeded multiple approximate units, other seeds such as basic logic gates with limited logic space were used to develop completely new multi-bit approximate adders with good fitness levels.

The effect of process variation was also calculated. As the number of transistors is reduced, the distribution of the transistor widths and gate oxide may shift away from a Gaussian Curve. This result was demonstrated in different types of single-bit adders with the delay sigma increasing from 6psec to 12psec, and when the voltage is scaled to Near-Threshold-Voltage (NTV) levels sigma increases by up to 5psec. Approximate Arithmetic Units were not affected greatly by the change in distribution of the thickness of the gate oxide. Even when considering the 3-sigma value, the delay of an approximate adder remains below that of a precise adder with additional transistors. Additionally, it is demonstrated that the GA obtains innovative solutions to the appropriate combination of approximate arithmetic units, to achieve a good balance between energy savings and accuracy.

Dedicated to my father, who was the rock behind my back

ACKNOWLEDGMENTS

The author wishes to express his gratitude to Dr. Ronald DeMara, advisor and committee chairman, for his technical guidance and unlimited support.

The author would like to thank the thesis committee members for all of their guidance through this process.

Appreciation is also extended to Rizwan Ashraf for his technical assistance.

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CHAPTER 1: INTRODUCTION

As environmental concerns and portability of electronic devices move to the forefront of priorities, it is important for us to find ways to reduce the power consumption of our devices. A number of ways have been proposed to reduce the power consumption of devices. The need for higher energy efficiency in recent times has pushed the industry and academia to find various other avenues to decrease power consumption. Some of the common methods of reducing energy consumptions include voltage scaling, Probabilistic CMOS (PCMOS), algorithmic truncation and approximation. These methods are reviewed briefly here. Various mechanisms are used to reduce errors due to voltage overscaling, one of the mechanisms of particular interest is shown in [9] . In [9] slack redistribution is used to gracefully degrade the circuit. Use of PCMOS is another way of reducing the energy cost without compromising on accuracy by a large margin as shown in [10]. [10] states that all MOS devices are inherently probabilistic rather than deterministic, it takes advantage of this probability in cases where noise is present to yield accurate results. [10] Implements PCMOS in System on Chips (SoC) to form a Probabilistic System on Chip (PSoC). The third method i.e. algorithmic truncation is a software level based solution, this solution is usually compared with hardware level approximation as shown in [7]. Algorithmic truncation usually refers to methods of removing a fixed number of LSBs from the output or input. Another method of algorithmic adaptation would be to skip certain steps in the algorithm in return for reduced accuracy and power savings. Hardware approximation refers to reduction in the size of critical path by reducing the number of transistors in the path.

The paper [15] illustrated the various ways a Genetic Algorithm can be parallelized, this paper delves into four levels of parallelism i.e. Parallelism of fitness evaluation, Parallelism of reproduction (selection, crossover etc.), Parallelism of groups (Population is sub-divided into groups and then they are evolved separately, with a chance of interaction after each generation), the paper

also defines the various levels of details a parallel genetic algorithm can be implemented. The second paper uses a Cartesian Genetic Algorithm to develop various adders at design-time, also the adders developed undergo testing under various error metrics, they also incorporate redundancy in their method where not all the gates are used for obtaining an output, a downside to this could be the area size, the paper does not describe how the other gates are excluded and how they can be included i.e. the method used. It uses two levels of fitness calculation to test the error range of the circuit. The third paper apart from developing their own parallel multiplier, they also compare various types of multipliers and where they could be used, they give design guidelines to use when implementing it in EDA tools.

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Significance of the Problem

For most purposes nature seems to approximate in many situations, for example the flow of a fluid, there are always a number of extraneous factors responsible for the flow of the fluid, unless the fluid is under ideal conditions, only then is it possible for science to truly predict the flow of a fluid. Signal Processing is similar to such a problem, analogous to a fluid, it is inherently tolerant to approximations and error. We try to exploit this inherent tolerance to errors to reduce Power consumption and area used by computation. Such kind of inherent tolerance can also be seen in Machine Learning, and Data Mining, which show significant algorithmic resilience [3]. This trait can be attributed to redundancies in input data-sets, lack of a definitive result or the ability of an algorithm to average out the errors. In general, most of the signal processing applications require high-end processing powers to compute precisely and expend large amounts of power to compute precisely. Approximation could save a lot of power in applications where 100% accuracy is not

a necessity. As it removes the unnecessary slack of more than required accuracy using various methods such as voltage over-scaling, functional approximation etc.

We functionally approximate many arithmetic circuits to improve delay and power consumption in adders. We also observe the effects of process variation as illustrated in the tables further into the text. In combination with Genetic Algorithms(GA) we develop combinations of multi-bit adders with a threshold accuracy and major savings in power. The GAs help us find the right combination of accuracy and power savings, where the trade-off is just right such that the accuracy is not degraded too much and the power savings are maximum. Process variation also seems to play a significant role in smaller bit-width adders, but as the size of the arithmetic unit increases, the distribution evens out and the sigma value seems to remain within normal ranges in approximate units.

Types of Approximations

Approximations can be broadly classified as *over-scaling based approximation* methods which introduce errors due to timing constraints, some examples of over-scaling based approximations include Voltage Approximation. Usually in over-scaling based approximations the slack time is progressively decreased until there's not enough time for the circuit to latch on to the value, which causes the circuit to use the last value latched as the current value, in voltage approximation, the V_{DD} or V_{TH} is progressively reduced until such temporal errors occur. For the sake of simplicity we include Probabilistic Computing in this category and *functional approximation* methods, which reduce the number of transistors in arithmetic units to introduce errors, maximum power savings are usually obtained by functional approximations. This is due to the fact that the parasitic capacitances, and dynamic, and static power consumption are drastically reduced as the number of transistors reduces, as shown in the results. A beneficial side-effect of reduced transistors is that

the delay also decreases significantly, process variation does not seem to affect the delay enough to increase the delay and power consumption to equal that of an accurate unit. Another type of approximation is known as *algorithmic truncation*, in this method a part of the code or number is truncated, in case of code truncation, a for loop could be truncated by reducing the number of times it needs to run, for example if a for loop needs to loop 1000 times to obtain an accurate result, it could instead run for 200 loops and get a nearly accurate result in case of binary truncation a part of the binary number is removed usually a set number of LSBs and replaced by 0's, for example if a binary number 11111111_2 is to be computed on instead of the hardware receiving 11111111_2 it receives 11110000_2 and the hardware performs the arithmetic operation on the first four most significant bits. The three examples of voltage over-scaling, Probabilistic Computing and functional approximation are explained further in the following sections.

Voltage Over-scaling

Orthodox designs take into account various redundancies to select a high V_{DD} to accommodate any errors occurring due to process variation, timing errors, aging etc. However, when V_{DD} is scaled below the critical voltage, the number of errors drastically increase and hence in turn drastically degrade the output quality. One way to improve signal quality in such cases is to introduce error correcting mechanisms. [11] introduces an approach under algorithmic noise tolerance. This type of circuit contains two blocks namely the main computing circuit and an error correcting circuit. The main computing circuit is run on a low V_{DD} whereas the simpler error correcting circuit runs on a higher voltage. As the error correcting circuit consists of lesser number of transistors than the main computing circuit due to which the energy savings are notable. In certain cases Error Correcting Codes (ECC) are used to compensate for the errors caused by over-scaling.

Probabilistic Computing

As CMOS technology scales down to the 10nm region, hurdles due to noise pose several challenges as explained in [10]. Probability based CMOS technology looks upon noise as a resource which can be harnessed [12]. Probabilistic CMOS have been known to yield better Energy Performance Product (EPP). EPP is defined as the product of energy consumed and running time. In [10] PCMOS is defined as the CMOS which is affected by thermal (ambient) noise. The computation in such circuits use probabilistic algorithms to reduce the effect of noise in the output. In addition to the above qualities, PCMOS can also generate high quality random bits. These random bits are important, as they are used extensively in probabilistic algorithms. The circuits implementing PCMOS typically contain a deterministic host processor, which is used to compute most of the control intensive components of an application and a probabilistic co-processor, which is used as an accelerator.

Functional Approximation

The process of removing transistors from a circuit to obtain a less than accurate circuit can be defined as Functional Approximation. The transistors removed are usually from the critical path to reduce the delay along with power consumption, but if the critical path is untouchable especially in cases where the MSB could be of significant importance, the transistors are removed from other areas which affect LSBs more often than MSBs, functional approximation gives us the most savings, as it reduces the cascading effect most times, removes a number of unnecessary capacitances present in the accurate circuits. In this thesis we concentrate on adders, we functionally approximate a number of adders including the CLA Adder and Ripple Carry Adder and determine their delay under various conditions and process variation is also taken into account in these conditions.

Evolutionary Algorithms

A number of evolutionary algorithms exist to obtain the optimal solution like Genetic Algorithms, gene expression programming and Differential Evolution . Almost all of these follow a general pattern as shown in Figure 1.1. Most Evolutionary Algorithms usually consist of an initial population on which genetic operations such as mutation and crossover are performed. After the genetic operations are performed the viability of the solution is evaluated this step is known as fitness evaluation. This process is done until it has reached the required solution or when the requested number of loops has reached.

```
1: procedure GENETIC ALGORITHM(a, b)                                ▷ population consisting of a and b
2:   while a & b > error constraint do
3:     population allocation
4:     genetic_operators()
5:     fitness_evaluation()
6:   end while
7:   return c & d                                                  ▷ c & d are the latest generation
8: end procedure
```

Figure 1.1: Evolutionary Algorithms

In our research we use Genetic Algorithms (GA) as an optimization technique to obtain the best combination of power savings and accuracy. One of the advantages of the GA is its ability to escape from a local minimum to global minimum, as shown in Fig. 1.2. This is mostly due to the mutation factor, where a random mutation can flip the GA into the path of finding the global minimum. Although it is not a certainty that the GA would find a globally optimum solution, there is a high probability that the GA would find the global minimum.

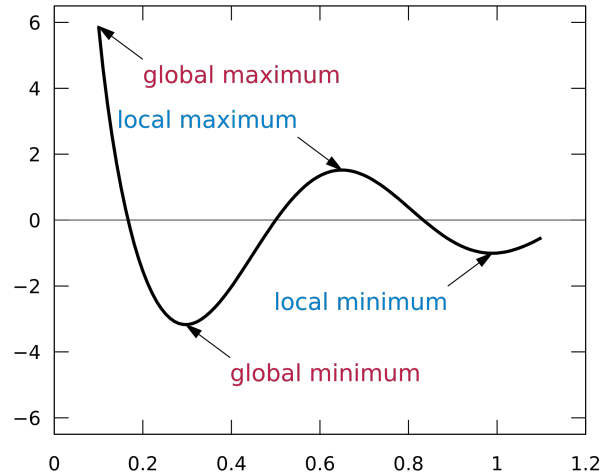


Figure 1.2: Solution Space [1]

Genetic Algorithms

Fundamentals of Genetic Algorithms

This section reviews some of the fundamental terms of the evolutionary-based computing approach known as Genetic Algorithms.

A GA, at the genesis of the procedure contains a number of random individuals generated by the system or pre-seeded by the user, depending on the extent of automation desired. The individuals together create a *population*. When this population is manipulated over time, it defines a sequence of *generations*, whereby the individuals of a population react with each other using Genetic Operators to form new individuals, these newly-created individual designs are referred to as *offspring*, and the population of offspring is a generation. Usually the original population is denoted as zeroth generation. Every individual in the population can be represented as a binary string, which is referred to as the *chromosome* of the individual. The chromosome is an important data structure which determines the design characteristics including device selection and the interconnection of

the individual circuit being represented. The performance of an individual can be evaluated by using a *fitness* function. Fitness can be specified as a cost function including mean error and resource usage according to a user-defined value called the *error criterion*.

As the GA runs, it performs Genetic Operators such as a *crossover*, which is a method by which interacting individuals exchange binary data, replacing the original data, to exchange subcircuits from high-performing parents to offspring. To make sure that the GA does not converge to a local solution, a *mutation* function is introduced, which randomly changes a value in the chromosome. The probability of mutation is kept to low levels in the range of one random modification for every 1000000 bits. The mutation operator is kept to such low levels to prevent the GA from performing a random search. In certain GAs, the fittest individual of each generation is always retained through successive generations without any modification, this process of selection is referred to as *elitism*. In cases where elitism is not used, individuals are selected based on probability, which is a function of its fitness. If the fitness level of an individual is high, then it has a higher chance of being selected and vice-versa.

Parallelism in GAs

The power of GAs is given by schema theorem [13], the schema theorem is a set of symbols used in a string $\{1^*00^*1\}$ where $*$ can either be one or zero. The order of the schema $o(H)$ is defined as the number of fixed positions in the schema, while $\delta(H)$ is the distance between the first and the last specific positions. The order of the string 1^*00^*1 is four and the defining length is five. The schema theorem states that the fitness of a short schemata with higher fitness than the average population will increase exponentially in successive generations [14]. It is expressed by the equation:

$$E(m(H, t + 1)) \geq \frac{m(H, t)f(H)}{a_t}[1 - p] \quad (1.1)$$

where, $m(H, t)$ is the number of strings belonging to schema H , $f(H)$ is the average fitness of schema H , a_t is the average fitness at generation t and p is the probability that a mutation could destroy a schema. Here p is defined from the following equation:

$$p = \frac{\delta(H)}{l-1}p_c + o(H)p_m \quad (1.2)$$

where, $o(H)$ is the order of the schema, l is the length of the code, p_m is the probability of mutation and p_c is the probability of crossover [14]. From equation (1) and (2) it can be inferred that it is advantageous to keep the chromosomes as short as possible. Also, from the schema theorem it can be inferred that even in a sequential algorithm, it is possible for the algorithm to handle $o(N^3)$ schema at a time for a population of N individuals. Thus the GAs can handle large number of schema at a time, which can be considered as inherent parallelism [13].

Most GAs can be subdivided into four parts based on amount of computation namely:

- Population Selection
- Crossover and Mutation
- Fitness Evaluation
- Offspring Generation

The list also illustrates where a GA could be parallelized with greater efficiency overall. Apart from this a GA can also be parallelized based on its detailed algorithmic flow [13]. Some of the methods mentioned in [13] are

- *Parallelism of Individual's Fitness*: The fitness evaluation of a population consumes the most amount of power and time, depending on the algorithm and fitness dependencies, the fitness calculation can be parallelized.

- *Parallelism of Occurrence of Offspring*: Processes by which offspring are produced could be parallelized, for example, when a crossover takes place only two parents are used in a single crossover, while the rest of the population lies idle, this can be rectified in parallel computation by performing crossover simultaneously over the entire population.
- *Parallelism Based on Population Grouping*: In certain cases, the population of individuals could be subdivided into smaller sub-groups which could be assigned to different threads.

A number of methods have been proposed to quantify the errors occurring in approximate circuits, some methods are similar to Hamming Distance. In this thesis we review three types of error measurement techniques as mentioned in [2].

Error Distance

The measurement technique developed [2] measures the precision of approximate circuits. This method is termed as *Error Distance*. Error Distance is the arithmetic distance between two binary values i.e. the arithmetic distance between the predicted/required value and the obtained/output value. For example, let us consider a Full Adder, if the exact output of the Sum is 100101, but the circuit gives an output of 100100, then the error distance between the two values is said to be one, similar to Hamming distance. But if the output was 101111” then the error distance would be ten. It can be seen that the value of error distance (ED) increases or decreases depending on the position of the incorrect bit. [2] defines ED with equation(1.3)

$$ED(a, b) = |a - b| = |\sum_j a[i] * 2^i - \sum_i b[j] * 2^j| \quad (1.3)$$

For a non-deterministic application, the output is probabilistic and usually follows a set distribution for a given input a_i . In this case ED is defined as the weighted average of all

possible outputs to the nominal output. Assume that for a given input, the output has a nominal value b , but it can take any value given in a set of vectors b_j ($1 < j < r$) [2]. The ED of the output, in such cases, is given by equation (1.4) as shown in [2]

$$d_i = \sum_j ED(b_j, b) * p_j \quad (1.4)$$

where, p_j is the output probability of b_j ($1 < j < r$)

Mean Error Distance

Mean Error Distance is used in cases where the inputs are probabilistic and thus each input occurs only at a particular probability. The mean error distance (MED) of a circuit is defined as the mean value of the EDs of all possible outputs for each input [2] . Assume that the input is a set of vectors a_i and that each vector occurs with a probability of q_i [2]. Then, the MED, d_m of the circuit is given by

$$d_m = \sum d_i * q_i \quad (1.5)$$

Where, d_i is the ED of outputs for input a_i which can be computed by (1.5).

As an example assume a NOT gate, the probability of the input to the NOT gate being 1 is 0.7 and if the output of the NOT gate is 1 even if the input is 1. In this case the MED is given by ED multiplied by the probability (0.7) which is equal to 0.7 as the ED is 1 in this case.

In case of multiple outputs, the ED is individually calculated and then multiplied with individual probabilities and after the values are calculated, all the values are summed with each other to obtain the final value.

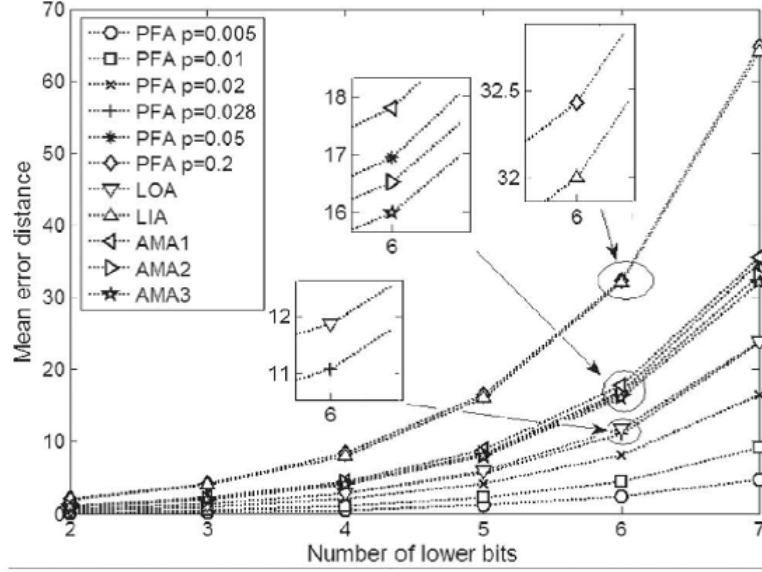


Figure 1.3: Normalized error distance versus the number of lower bits [2]

Normalized Error Distance

MED increases with increase in the number of lower bits. It is therefore biased to use MED to compare between two adders with different lower bits as the maximum value of error that can be effectively reached has also changed [2]. To overcome this limitation *Normalized Error Distance* (NED) is used. NED is defined as

$$d_n = d_m / D \quad (1.6)$$

Where, d_m is the MED and D is the maximum value of the error. The maximum value is usually $2n$ for n lower bits [2]. We use the fitness function used in [15]. It requires multiplication of the number of combinations possible in a truth table multiplied by the number of outputs to get the maximum fitness value. Hence, we can denote the formula as given in equation (1.6).

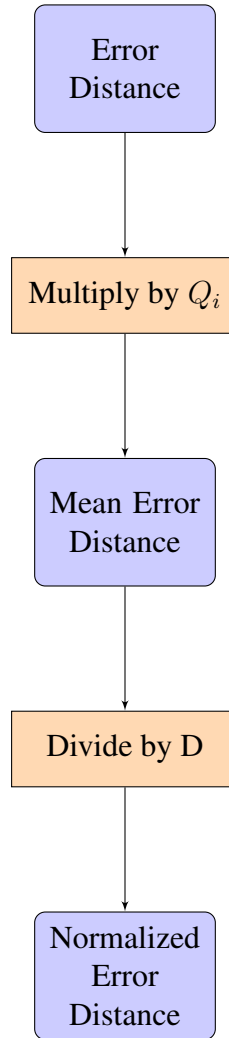


Figure 1.4: Relation between error metrics

Challenges facing Evolution of Approximate Adders

Currently the important facet of a GA is the fitness evaluation, it is also the most computation intensive task of the GA. One of the problems we have tried to tackle here is the evaluation time of complex solutions. In this case it was a multi-bit adder where the input space is enormous, in such cases we have used parallelism to generate the results faster by using multiple threads and efficient programming.

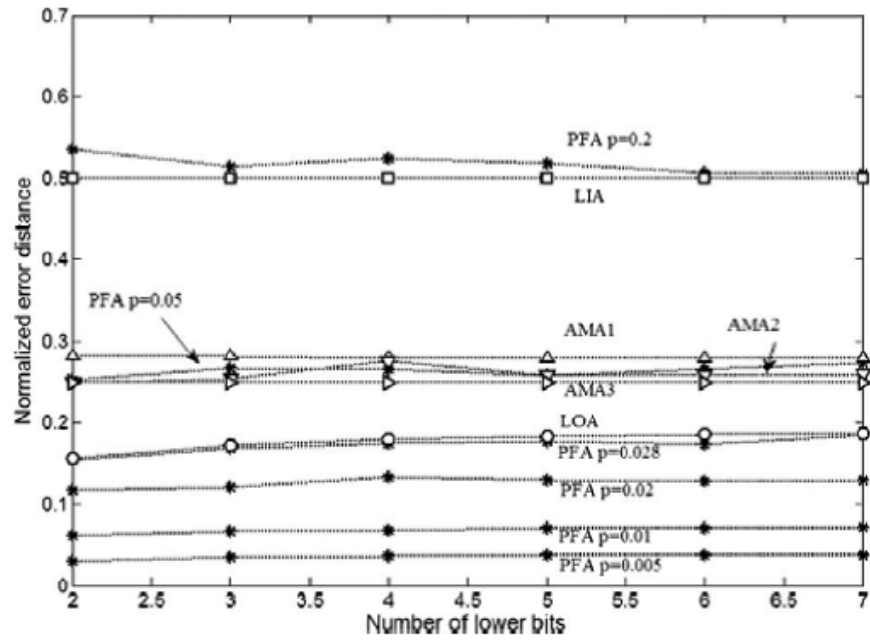


Figure 1.5: Normalized error distance [2]

Another problem is to generate sequential circuits using parallel methods due to which the interconnections are performed incorrectly. Additionally in some cases the GA would not converge to a globally optimum solution, in such cases the mutation factor needs to be set by trial and error method until a proper solution is obtained, this is an extremely time consuming process. Also GA can be further optimized by combining them with other optimization algorithms, where the coefficients can be altered by the GA or vice versa. Also some GAs face problems with random seeds, where the seeds aren't random enough for complex algorithms to take a different path at every run. Fortunately the population size is relatively low in our algorithm, hence the effect of pseudo-randomness is not pronounced. Some other challenges in implementing a GA is to identify portions of GA and areas where it could be potentially parallelized with gains, parallelizing at every opportunity does not yield good results, as in some cases the overhead of multi-threading is greater than the gains obtained from parallelizing a particular area. As such all the loops hence

were not parallelized as it was more efficient to run them on a single thread rather than multiple thread. Another issue faced was the placing of certain volatile variables in the cache, which were being constantly modified during runtime, these variables had to be manually identified and marked volatile to prevent invalidation of the cache line.

Contribution of Thesis

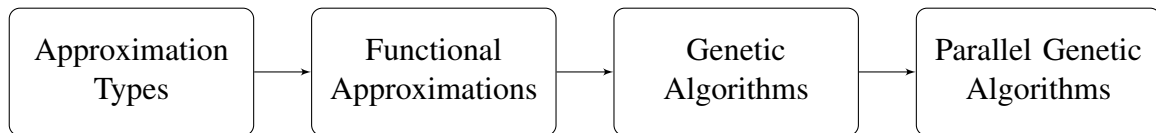
The contributions of this Thesis are:

- *Effect of Process Variation on Approximate Adders at NTV*: Determines the range of values possible for delays and power consumption under the effect of process variation and analyzes how it affects a larger system of approximate units,
- *Evolution of approximate systems using GAs*: Identifies various dependencies in Adder circuits and how it affects the performance of the GA and explores the various avenues through which the GA could be parallelized to obtain higher efficiency and lower process time, and
- *Development of Approximate Adders*: Develops a new approximate adder from a CLA Adder with better performance and lower delays for the sum output.

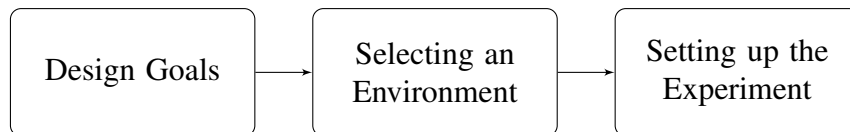
Organization of Thesis

The organization of thesis is depicted in Figure 1.6. First, related work is reviewed in Chapter 2. Chapter 3 defines the research and experimental methodology which was employed, including GA configuration for the approximate adder circuits being evaluated. Chapter 4 presents results for variation in RCAs and CLAs. It also discusses lessons learned for using parallel GAs to optimize these circuits.

Chapter 2: Related Work



Chapter 3: Methodology



Chapter 4: Results

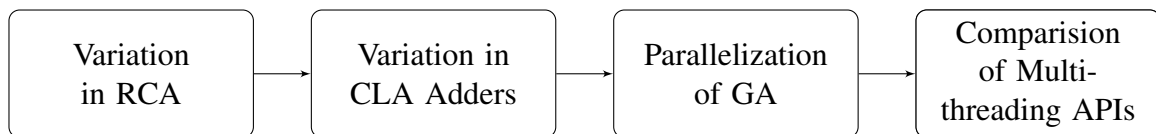


Figure 1.6: Organization of Thesis

CHAPTER 2: RELATED WORK

Approximate computing offers a promising approach for reduced energy operation for applications which can tolerate some imprecision. There are many types of approximate circuits, including those constructed by voltage over-scaling and over-clocking [16] and others as mentioned in [17–20]. However, this thesis uses approximate adders which utilize lesser number of transistors than the original accurate design. The Adders used in this thesis are obtained from [7]. As these adders are analyzed in a manner which is suited to the application presented in our research. In this section we discuss and analyze the three different types of adders used and the accurate adder. The accurate adder is the reference adder from which the other adders are derived.

Types of Functional Approximation

A number of methods to approximate circuits have been proposed including the trial and error method. Most of the approximations have been done manually, carefully reducing the number of transistors in a circuit using the trial and error method. Most of the efforts concentrate on reducing the length of the critical path. Some other methods such as SALSA [3] and MACACO [4] use automatic synthesis using iterative methods. In [3], the authors construct a Quality Constraint Circuit (QCC) as shown in Figure 2.1, which is used to construct the approximate circuit. The QCC contains three circuits, the original accurate circuit, the developed approximate circuit, and the quality function circuit, which compares the output of the approximate circuit and the accurate circuit and determines if the error of the approximate circuit matches with given quality constraint. If the quality constraint is met i.e. the output of the quality circuit is '1', the program iterates through don't cares, removing each don't care whenever the output of the QCC is 1. When the QCC gives an output of '0' i.e. in a situation where the approximate circuit doesn't meet quality

constraints, the platform reverses to its last valid state and this last output is considered as the valid situation.

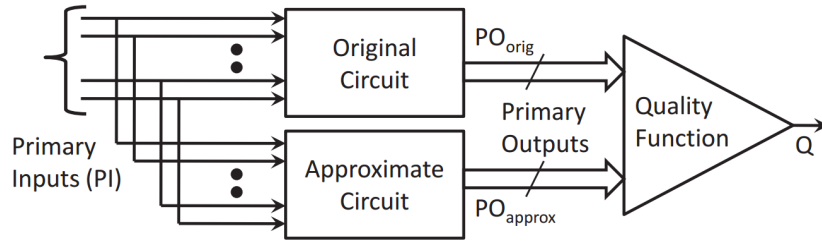


Figure 2.1: Quality Constraint Circuit [3]

In [4], the authors develop an automated method to check if the approximate circuits developed meet the requirements set beforehand. In [4] the functionality of the circuit was determined by overscaling the circuit, after obtaining the results from the over-scaled circuit, they create a platform which functionally approximates the original circuit to meet the over-scaled circuit's results. This results in better power savings and delay, but this new circuit is equivalent to an un-timed circuit of the same output. Next a virtual circuit for error analysis is constructed by instantiating the equivalent un-timed circuit and the golden circuit. The outputs are then compared to quantify the error. This virtual circuit computes the error between the developed approximate and golden circuits for a given input set. A overview of the methodology used in [4] is given in Figure 2.2

In [2], the authors create two approximate adders by manually reducing the mirror adder. The authors create a *lower part OR adder*, which uses OR gates for generating lower bits and an AND gate to generate carry-ins for the more significant bits. The lower part OR adder (LOA) neglects the carries of the LSBs, which results in the loss of precision. The adders developed by the authors shows an improvement of upto 25% in dynamic power consumption. [5] proposes an accuracy configurable multiplier, the multiplier makes use of redundant arithmetic units to get accuracy on

demand. The authors in this paper have concentrated on improving the accuracy of the lower bits instead of the higher bits, the authors claim that the more significant bits are largely accurate, and the remaining bits are inaccurate. This paper divides the multiplier circuit into multiple parts an accurate part and an inaccurate part. The paper then uses a carry-in prediction logic developed by the authors to make the final output's most significant bits accurate. The recursive method used in [5] is shown in the following figure.

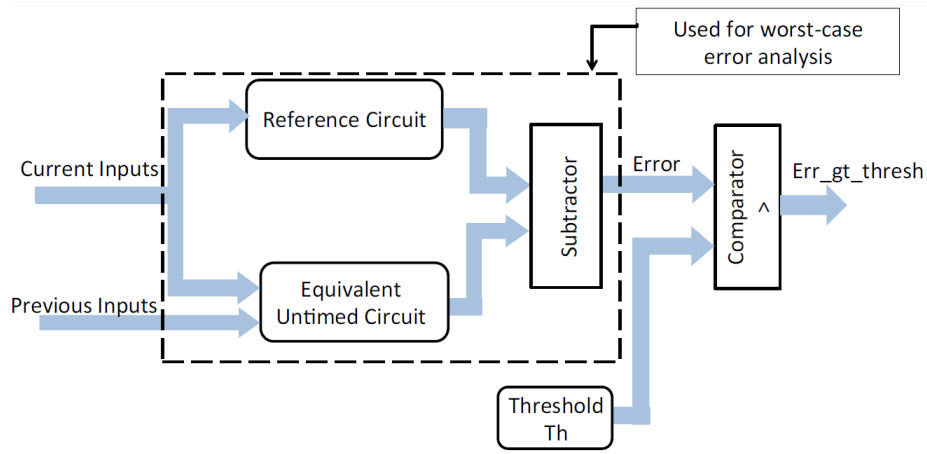


Figure 2.2: Quality Constraint Circuit [4]

Kahng et. al in [6] reduced the length of the carry chain to approximate the adders and reduce the delay. The authors introduce an extra sub-adder to increase the accuracy of the unit when required. The authors also propose a error correcting mechanism where absolute accuracy is needed.

This error correcting mechanism has an overhead in power and area. The error correction and detection mechanism used in [6] can be seen in Figure 2.4. [6] also implements this system for pipelined architecture as shown in Figure 2.5.

For the experiments performed in this research we used the adders from [7], which are explained in detail in the following sections.

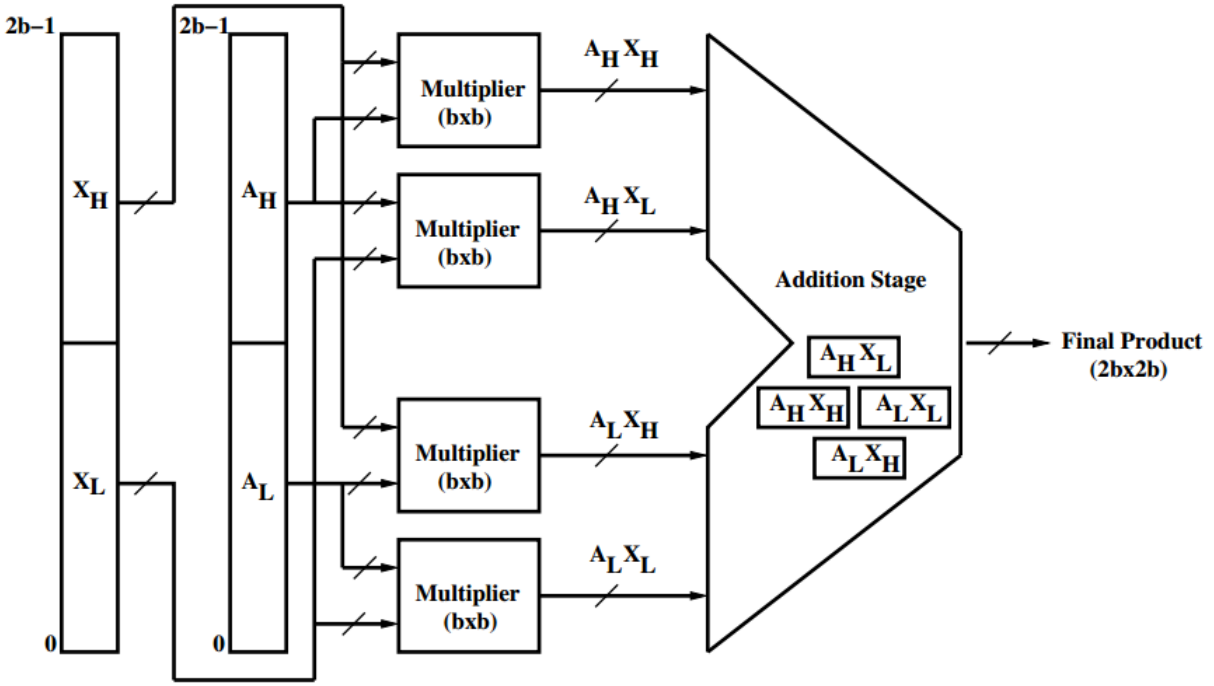


Figure 2.3: Recursive multiplier used in [5]

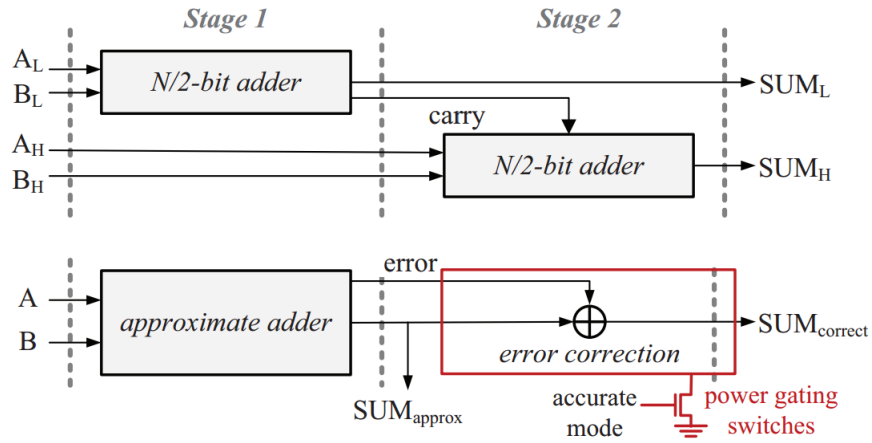


Figure 2.4: Pipelined adder implementation of accurate adder and approximate adder used in [6]

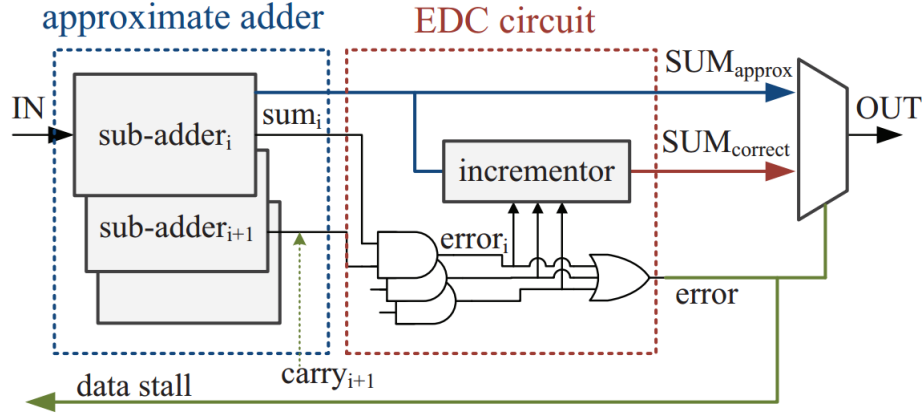


Figure 2.5: Error Detection and Correction Mechanism used in [6]

Conventional Mirror Adder

A Conventional Mirror Adder (CMA) consists of a total of 24 transistors [7], as shown in Figure 2.6. The other approximate adders are obtained from the CMA.

Approximation 1

The first approximate mirror adder is obtained by reducing the number of transistors in the schematic one by one until the defined error constraints are breached. Once the error constraints are breached the last modification is reverted to obtain the approximation [7], this method is applied to the other approximate adders as well. AMA1 is shown in Figure 2.7.

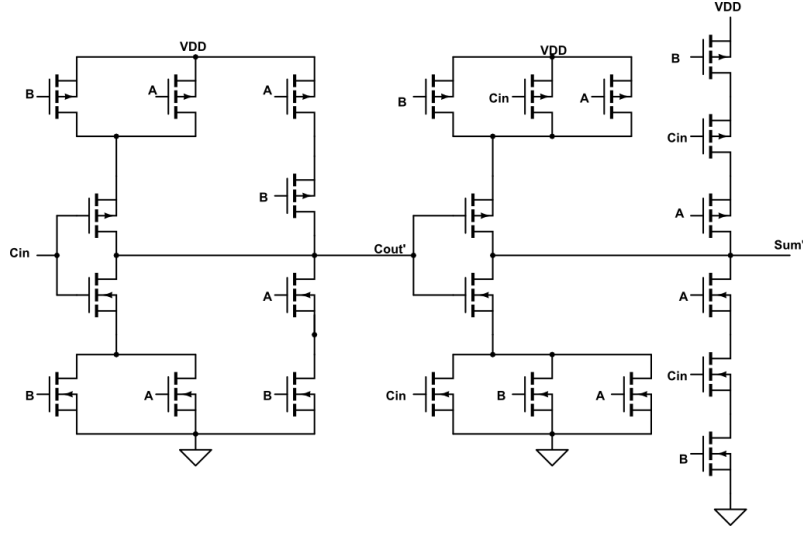


Figure 2.6: Conventional Mirror Adder [7]

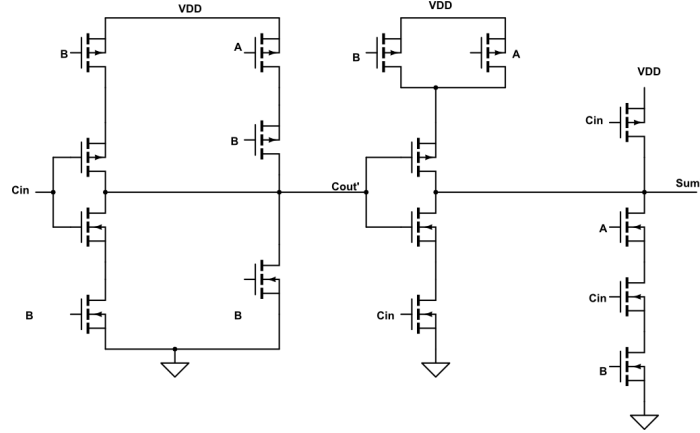


Figure 2.7: Approximate Mirror Adder 1 [8]

Approximation 2

Approximate Mirror Adder 2 (AMA2) introduces two errors in C_{out} and three errors in Sum , as shown in Table I.

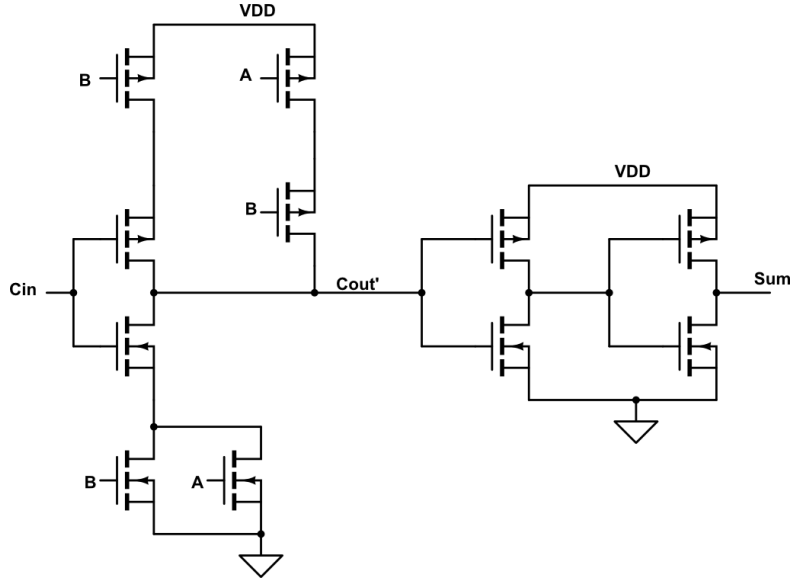


Figure 2.8: Approximate Mirror Adder 3 [8]

Approximation 3

Approximate Mirror Adder 3 (AMA3) C_{out} and Sum are accurate for four out of eight outputs, the schematic for AMA3 is given in Figure 2.8.

The performance in terms of boolean values can be determined by using the truth table shown in Table 2.1. These values are later on used in a GA to test the fitness values of the developed multi-bit adders.

Energy Evaluation

In this thesis, we define error based on the accuracy of the output, we do not consider delay, although all the approximate adders here perform much better in terms of delay and power when compared to the fully exact adder.

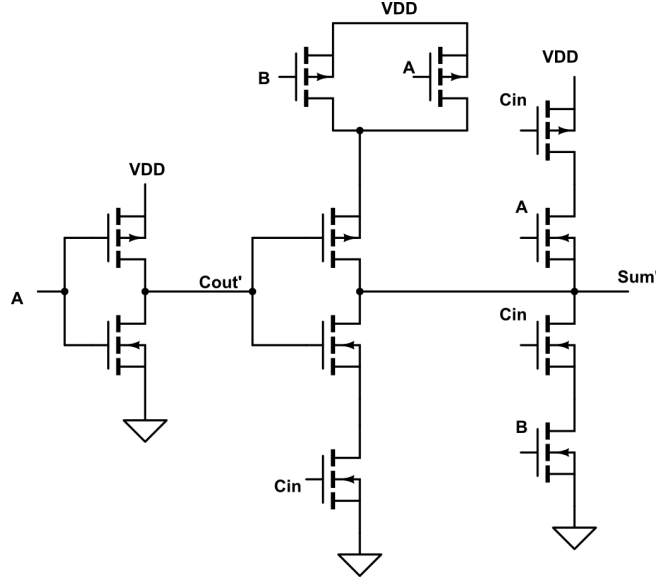


Figure 2.9: Approximate Mirror Adder 4 [8]

Table 2.1: Truth Table for Conventional Full Adder and Approximations 1, 2 and 3 [7]

Inputs			Accurate Outputs		Approximate Outputs					
A	B	C_{in}	Sum	C_{out}	Sum_1	C_{out1}	Sum_2	C_{out2}	Sum_3	C_{out3}
0	0	0	0	0	1	0	0	0	0	0
0	0	1	1	0	1	0	1	0	0	0
0	1	0	1	0	0	1	0	0	1	0
0	1	1	0	1	0	1	1	0	1	0
1	0	0	1	0	1	0	0	1	0	1
1	0	1	0	1	0	1	0	1	0	1
1	1	0	0	1	0	1	0	1	1	1
1	1	1	1	1	0	1	1	1	1	1

The dynamic power dissipation of the system is given by equation (7) and the static power dissipation is given by equation (8). Although power dissipation is not explicitly used in evaluation it enables us to understand where the transistors could be removed to maximize power savings. Additionally dynamic power dissipation which is currently gaining prominence in academia can

be considered in addition to static power consumption for the approximation of the circuits.

$$P_D = C_{pd} * V_{CC}^2 * f_I * N_{SW} \quad (2.1)$$

Where,

P_D is the dynamic power consumption

V_{CC} is the supply voltage

f_I is the input signal frequency

N_{SW} is the number of bits switching

C_{pd} = dynamic power-dissipation capacitance The total static power consumption of this device can be given as:

$$P_S = V_{CC} * I_{leakage} \quad (2.2)$$

Where,

P_S is the static power consumption

V_{CC} is the supply voltage

$I_{leakage}$ is the current into a device (sum of leakage currents)

From, equation 2.1 and 2.2, we can derive equation 2.3,

$$P_T = \Sigma(P_D + P_S) \quad (2.3)$$

The next factor we consider is the average propagation delay of an adder which is given in equation (2.4).

$$t_p = \frac{t_{PHL} + t_{PLH}}{2} \quad (2.4)$$

Where,

t_{PHL} is the time delay from High to Low

t_{PLH} is the time delay from Low to High

In chapter 2 we discuss previous work associated with the experiments performed in this thesis. We also discuss the various formulas used in the analysis of the performance of the approximate adders developed in this thesis and other related works.

CHAPTER 3: METHODOLOGY

Process variation in the described adders is simulated on HSPICE. The process variation is simulated by varying the threshold voltages. The values for threshold voltage follows the Gaussian curve shown in the figure below. The hypothesis is that when the number of transistors is reduced in a system the Gaussian curve tends to become flat.

The adders were translated into HSPICE language. The library files used were for 22nm process CMOS. These files were created by 3rd party at the PTM website created by Arizona State University. The 22nm CMOS used in this experiment were based on the latest Intel models using the FinFET technology. The library files were modified to accommodate the various threshold voltages obtained by Monte Carlo simulations. The Monte Carlo simulations were done over a thousand times to obtain the mean value of the delay and power consumption of the accurate and approximate adders. The values for the threshold voltage for each transistor was randomly picked from each run of 5 times in addition to a 1000 time simulation of all the values, also each nMOS and pMOS device was given a different threshold voltage even within the same system. This system was used to recreate an environment closest to that of a real world system. After the results were obtained the means for all the parameters were calculated to give a fair idea about the results versus the results obtained at different values.

The modification of the HSPICE values were performed for every single simulation. This could be made possible using the line of code `dev_vth0 = agauss (0, abs_val, 1)`, this gives us various values within the range of the values specified. Writing the above line of code does not guarantee Monte Carlo analysis, apart from this we also need to include DC sweep analysis so that HSPICE uses the values obtained above during simulation. Another option which needs to be used to make sure that each of the transistors in the system are simulated with different values from the Monte Carlo simulation is to use the line of code `modmonte = 1`.

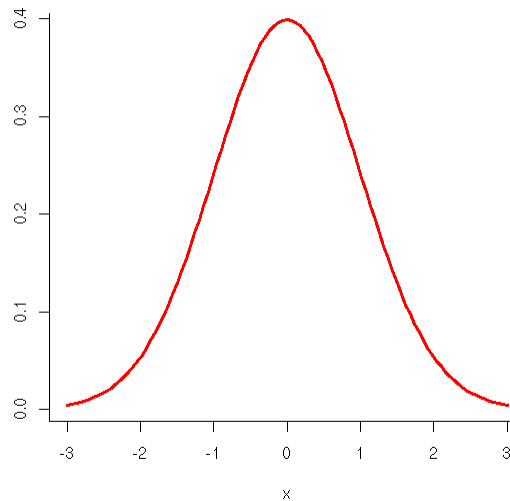


Figure 3.1: Gaussian uniform distribution

Once the circuit is translated into HSPICE code and all the Monte Carlo simulations are performed the .lis files obtained from the simulation are used to obtain the values for delays for Sum and Carry. The .lis file also contains the mean values of the 1000 Monte Carlo simulations performed. The number 1000 was chosen to give us a statistical probability of matching the real world setup at 99.9%. The mean values are noted down, along with the calculated sigma values, these values are then analyzed to determine if they conform to our hypothesis.

Experimental Results

The GA used to find the optimal solution is entirely written in C++. The program takes its input from a .txt file generated from another program. The other program is standalone and does not depend on the main program. This secondary program is used to generate the truth table which is used as input to the main program. This program can produce truth tables of varying lengths

depending on the input bit-width. This program is used because it would be a tedious process to create the truth table on a 8 bit adder manually.

The program used here tries to mimic an FPGAs architecture which includes CLBs, LUTs etc. The GA constructs its chromosome from the available resources, and the original population is instantiated, with each LUT assigned to a random individual. The LUT in the program is instantiated as a class, within another class. Each LUT object consists of three vectors and a variable to store its function type. The vectors of the LUT object map its outputs and inputs, all of the inputs and outputs are binary values.

The LUT class is in turn instantiated in the individual class, the other vectors store the input and output values. The function *CalculateFitness* calculates the fitness of the individual by going through each LUT and compares the port values to the value given in the input golden truth table. the fitness value is incremented by one for each value matching the original golden output. For example, an individual representing a circuit with y inputs and x outputs will have a maximum fitness value of $2^y * x$. If the fitness of the individual is at a maximum, it indicates that all the outputs of the individual are perfect. The individual class is not the primary class, it is superseded by a larger class called the generation class.

The generation class contains the current population of individuals. The generation class consists of a *Crossover* function, depending on the prescribed rate of crossover, it selects two individuals at random and randomly picks a crossover point in a way that the LUT objects are not violated. The LUTs before the crossover point are from Parent A and beyond the point are from Parent B.

If the crossover fails to take place due to some random chance, either one of the individuals is directly copied on to the next generation.

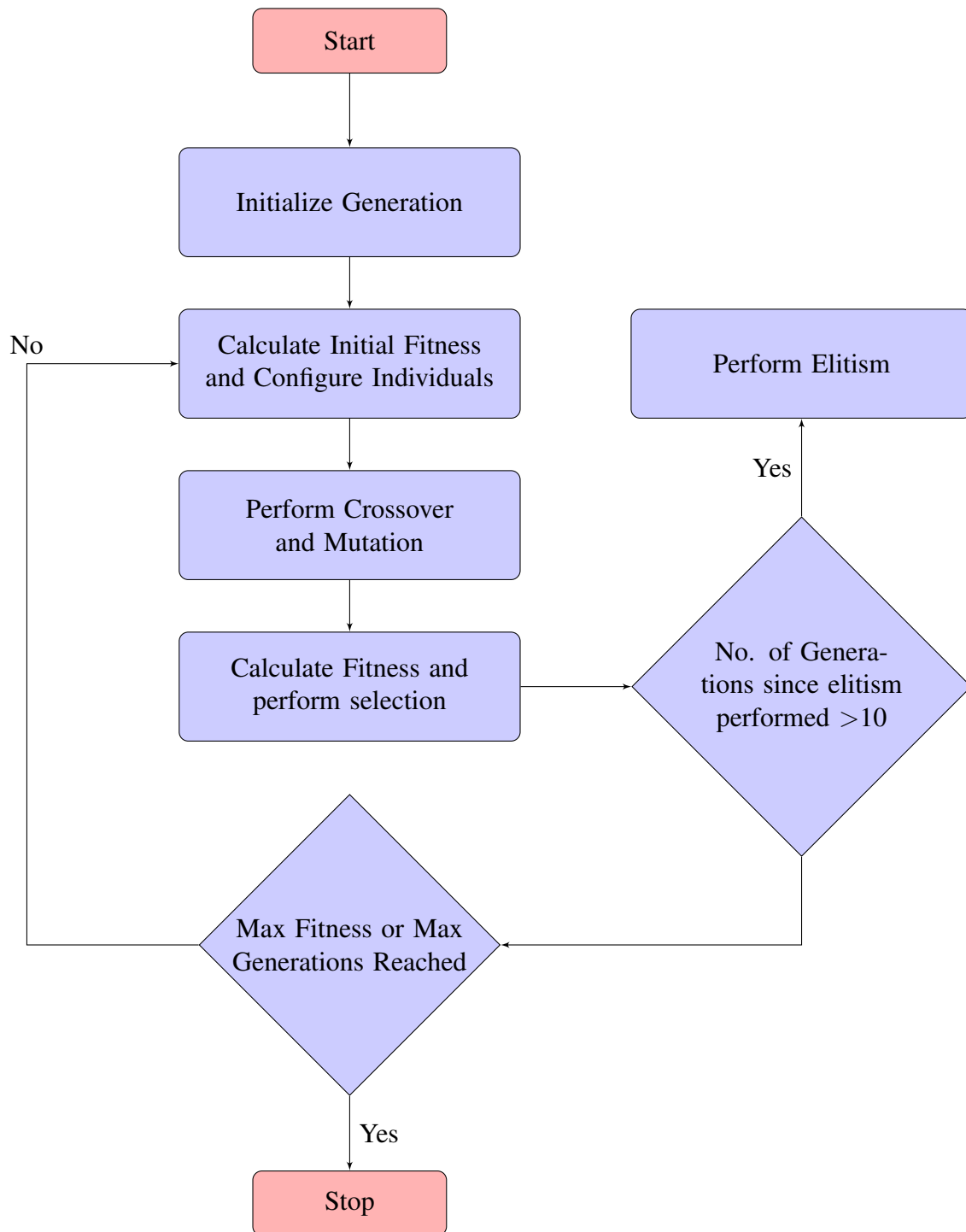


Figure 3.2: Flow of the GA

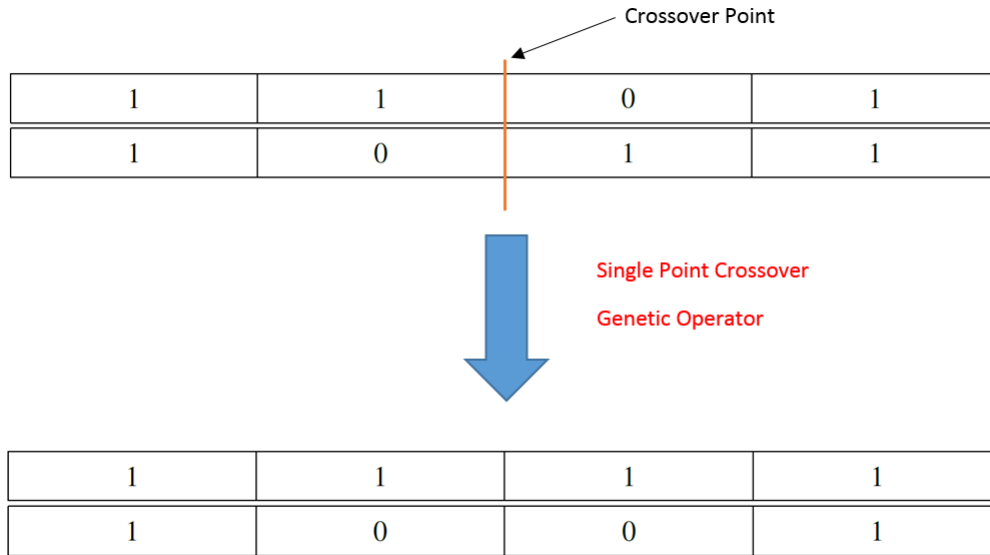


Figure 3.3: Crossover Operation

The next function, *Mutation*, performs a random mutation on an individual. The function chooses an individual and based on the user-defined mutation rate, assigns it a randomly chosen entity. Interconnection mutation wasn't performed because the Adders especially Ripple Carry Adders, which take their input from the previous single-bit adders do not form recognizable circuits when interconnections are randomly switched.

The *Selection* function randomly selects a defined number of individuals from the original parent population and the evolved offspring population. The individual with the highest fitness is moved directly to the next generation of individuals, irrespective if it was from parent generation or offspring generation. This process is repeated until the user-defined size of the generation is reached. The *DelayedElitism* function performs elitism every 10 generations to pick the best individual and stores it in a designated object for elite individual.

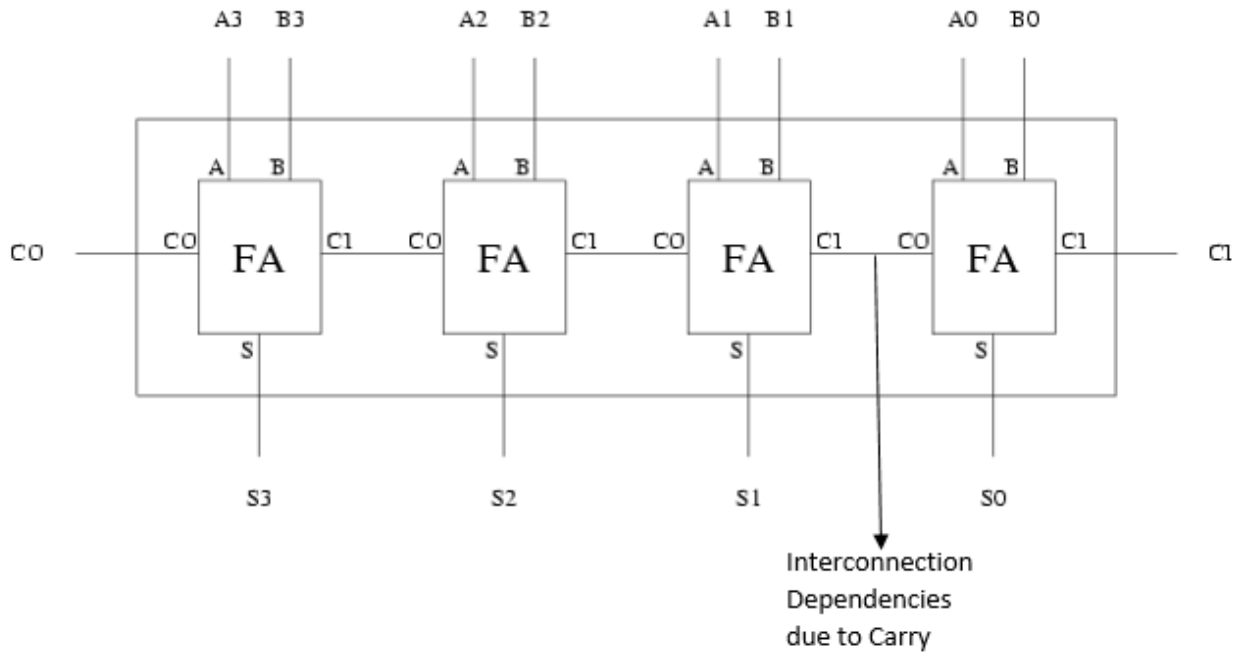


Figure 3.4: Carry Dependencies in RCA

Parallel Genetic Algorithm

Parallel Genetic Algorithms, henceforth referred to as PGAs can be subdivided into three groups based on the resolution at which the PGA works, namely

- *Global PGA (GPGA)*: A GPGA model uses the master slave method to divide the workload among threads. The slave threads calculate the fitness of an individual and perform genetic operations in parallel, whereas the master thread operates serially, assigning individuals to the slave threads. It is one of the most simple and effective PGAs to implement, but in GPGAs there is a high chance of uneven distribution of the workload. [13]
- *Coarse-grained PGA (CPGA)*: In a coarse grained model the population is sub-divided into groups and genetic operations are performed independently within a sub-population [13].

This type of PGA introduces a new function *Migration*. Migration defines the rate at which individuals from one group move from one sub-population to another sub-population.

- *Fine-Grained PGA (FGPGA)*: FGPGA assigns each individual to a thread and this individual interacts with another individual within a hamming distance of 1. This type of PGA could maintain diversity based on the neighborhood chosen. [13]

We implement a GA with three different modules: (1) Evolution of the circuit, (2) Selection of the offspring, and (3) maximum fitness calculation as shown in Figure 5. Each module consists of a for loop which loops through Look Up Tables (LUT) assigning different functionalities to each LUT based on a random number between zero to four each corresponding to AMA 1-4. The three for loops were parallelized using Intel TBB [21] *parallel_for* loop, also vectors were used wherever possible to give flexibility to the program. We also used OpenMP in conjunction with the Intel TBB library to test out how it affects performance relative to only using TBB, the outcome of this is illustrated in the results section. The main program was already designed by [15], we modified it to change the operation of the GA extensively. The original GA in [15] could only design MUX decoders, we changed the functionality of this Algorithm by modifying the LUTs used in the original program to map the different Approximate Adders as single LUTs. The LUTs considered in this program can handle 3 inputs and 2 outputs, analogous to a full adder. The LUTs are lowest grain level the program could go. The interconnection mutation was switched off for these genetic operations as this resulted in the two outputs being mapped to the wrong adder occasionally. The implemented algorithm produced designs which outperform the other adders, for example it obtains a fitness value of up to 40 for a maximum fitness value of 48.

The program uses a hierarchical approach; first the Individuals containing CLBs were designed, each CLB has a vector of LUTs. The individuals, CLBs and LUTs are initialized as classes as shown in [15]. The LUT class contains a function called *CalculateOutput* which takes in the input

vector to calculate the output based on the functionality chosen, this is done for each individual. The vector of individuals are initialized based on the initial population given in the input file.

We use a master slave method to generate workloads in the program. The workload is automatically distributed by OpenMP and Intel TBB. The compilation was done in Microsoft Visual Studio Express 2013. With the Intel TBB Plugin attached to the source of the program, as this is needed to parallelize the program, the number of threads to run the program is taken from an input file. This input file contains other information such as number of Configuration Logic Blocks (CLBs), number of Look Up Tables, number of LUT select Lines, number of LUT output lines, number of circuit input lines, number of circuit output lines, population size, mutation rate, crossover rate, number of elite individuals, number of generations, number of threads and number of runs.

Number of CLBs

This defines the number of Configurable Logic Blocks to be used in the simulation of GA.

Number of LUTs

This defines the number of Look Up Tables to be used in the simulation of GA.

Number of LUT select lines

This defines the number of Select Lines in each LUT to be used in the simulation of GA.

Number of LUT output lines

This defines the number of outputs in each LUT to be used in the simulation of GA.

Number of Circuit Input lines

This defines the total number of input lines for the entire circuit to be developed by the GA.

Number of Circuit Input lines

This defines the total number of input lines for the entire circuit to be developed by the GA.

Number of Circuit Output lines

This defines the total number of output lines for the entire circuit to be developed by the GA.

Population Size

This defines the total number of individuals initialized at the start of the GA.

Crossover Rate

This defines the probability at which two parents would be crossed over to produce offspring.

If crossover does not happen then the parents are directly copied to the next generation.

Mutation Rate

This defines the probability at which mutation would take place after the crossover is done.

Number of Generations

This defines the number of generations at which the simulation stops

Number of Threads

This defines the number of cores/threads to be used when the simulation is running

Number of Runs

This defines the number of times the simulation needs to be run.

In this chapter we identified the various dependencies in the Ripple Carry Adders and Carry Lookahead Adders and mapped them to the GA. These dependencies if not taken into account cannot be simulated on a computer program.

For Parallel GAs we looked at a number of API's including POSIX, OpenMP and Intel TBB along with their overheads when they are implemented. From the analysis we can conclude that although OpenMP and Intel TBB provide coarse grain control over multi-threading, they remove the burden

of the programmer to manually divide the tasks among threads, whereas OpenMP and Intel TBB do this automatically with greater efficiency.

CHAPTER 4: EXPERIMENTAL RESULTS

The four approximate mirror adders and the accurate adder were analyzed using HSPICE . Apart from simulation of process variation the adders were also subjected to varying Vdd voltages ranging from 0.65V to 0.8V (Nominal Voltage). The analysis was done on worst case scenarios where all the inputs were ones. In certain cases, the delay was not available because the output would not rise in such cases the inputs with the most number of ones which yielded an appropriate output such that the output delay could be measured. The results obtained for various adders are tabulated and explained here.

As V_{TH} variation does not affect all the Approximate Adders equally, then one of the ideas which can be implemented is logic diversity. Logic diversity involves using different sets of systems to gain the best overall performance.

In this case a GA could be used to involve a search space to determine which adders could perform best in a certain scenario. It is not always necessary for similar adders in a system to perform better than different types of adders in a system. For e.g. in a 16 bit Adder AMA-I and AMA-II may perform better in cases of DSP applications, but when only approximate calculations are required then a 16 bit adder containing solely AMA-I may perform better. In [22] soft resiliency has been mentioned, which can be defined as the property of a system to gracefully degrade the quality of the output as the components of a system malfunction. For a system to gracefully degrade, the system would require an underlying GA to perform an analysis of the search space and assign fitness value to each component during runtime. The fitness values can then be used to identify functioning components and use them instead of the malfunctioning items. Another method would be to use a partially malfunctioning LUT as an approximate gate. This could be especially useful in DSP applications where the loss of quality cannot be distinguished by humans.

RCA Adders

Table 4.1: Results of Accurate Mirror Adder

Voltage	Sum Worst	Carry Worst	Sum Mean	Carry Mean	Sum Variance	Carry Variance	Sum Sigma	Carry Sigma
0.8	148ps	58.09ps	131.46ps	43.41ps	8.36E-23	4.78E-23	9.14ps	6.91ps
0.75	152.8ps	66.04ps	133.82ps	48.76ps	1.05E-22	6.03E-23	10.24ps	7.63ps
0.7	160.4ps	79.45ps	138.48ps	55.61ps	1.52E-22	8.04E-23	12.32ps	8.96ps
0.65	173.3p	103.4p	145.1ps	65.83ps	2.83E-22	1.25E-22	16.81ps	11.19ps

Multi-bit approximate adder circuits are designed using the proposed methodology. Other benchmarks can also be designed if the multiple versions of approximate designs are available. For the benchmark utilized, the maximum fitness of the circuit is dependent on the number of inputs n and is equal to 2^{n+1} .

Table 4.2: Results of Approximate Mirror Adder I

Voltage	Sum Worst	Carry Worst	Sum Mean	Carry Mean	Sum Variance	Carry Variance	Sum Sigma	Carry Sigma
0.8	49.04ps	37.54ps	31.33ps	20.77ps	4.59E-23	6.83E-23	8.26ps	6.77ps
0.75	55.84ps	43.06ps	36.32ps	25.26ps	5.73E-23	8.35E-23	9.15ps	7.56ps
0.7	66.69ps	48.93ps	42.5ps	30.8ps	6.83E-23	1.08E-22	10.4ps	8.26ps
0.65	87.47ps	56.99ps	50.79ps	37.41ps	8.07E-23	1.85E-22	13.58ps	8.98ps

The GA was executed for a maximum of 5000 generations with a population size of 20 circuits consisting of randomly instantiated individuals. The GA selects a random set of individuals (parents) from the population after which *tbb_evolve* is invoked which evolves the circuit in parallel, which is followed by *tbb_postselect* and *maxfit* which are the processes for selecting the offspring and

calculating the maximum fitness respectively.

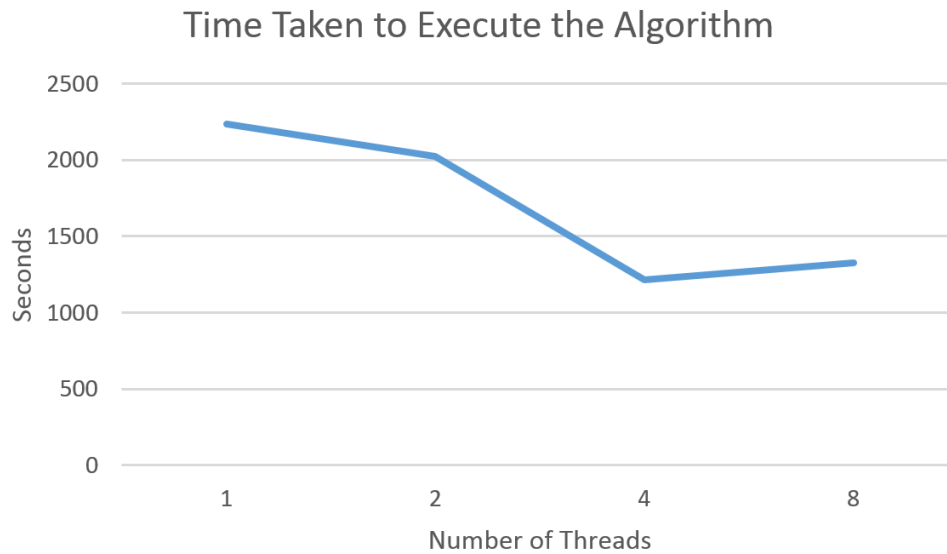


Figure 4.1: Performance without OpenMP

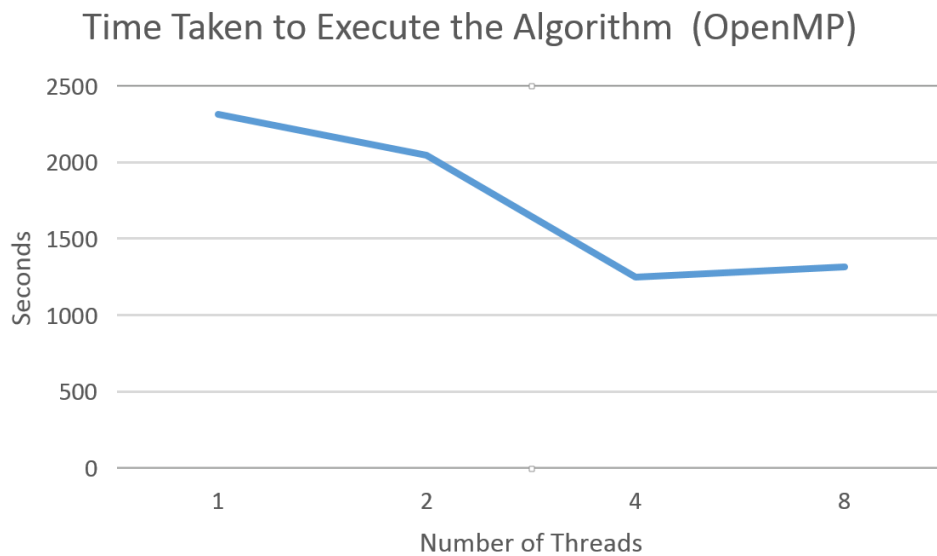


Figure 4.2: Performance with OpenMP included

One of the important observations was that the GA took as much time to evolve as to calculate the fitness, sometimes the time to evolve was greater than the time to calculate the fitness. This is a context-dependent observation as the time to evolve or to calculate the fitness values depends on the complexity of the calculation within each function.

Table 4.3: Results of Approximate Mirror Adder II

Voltage	Sum Worst)	Carry Worst)	Sum Mean)	Carry Mean	Sum Variance	Carry Variance	Sum Sigma	Carry Sigma
0.8	56.41ps	49.15ps	41.98ps	32.78ps	3.70E-23	2.83E-23	6.08ps	5.32ps
0.75	64.44ps	55.16ps	47.69ps	36.71ps	4.59E-23	3.52E-23	6.77ps	5.93ps
0.7	77.21ps	64.37ps	54.79ps	41.40ps	6.28E-23	4.45E-23	7.92ps	6.66ps
0.65	102.3ps	82.55ps	64.56	47.30ps	1.15E-22	6.77E-23	10.71ps	8.23ps

The most-intriguing result obtained was for a 2-bit adder including the carry with a fitness value of 40 out of the maximum fitness value of 48, also the MSBs were the least affected by the combination of approximate adders. Additionally, if a redundant adder is added then the fitness could be improved to 45, this is because the third adder could be better in certain cases, this could be useful when the output does not meet desired application criteria.

Even when a redundant adder is used or swapped with the current adder component, the power consumption would be less than that required for an accurate adder. The power savings may extend up to 63.8 % when compared to cases when an accurate adder is used in ideal conditions. For example if we consider a 2 bit adder composed entirely of accurate adders the total power consumption would be 517.9 nW as shown in Table II. In the same scenario if two approximate adders are to be used for example the AMA3 adder, then the power consumption would be 187.552 nW and if we add another redundant AMA4 adder, then the total power consumption would be 284.434 nW, which is still much lesser than the combined power consumption of the 2-bit accurate

adders.

Table 4.4: Results of Approximate Mirror Adder III

Voltage	Sum Worst	Carry Worst	Sum Mean	Carry Mean	Sum Variance	Carry Variance	Sum Sigma	Carry Sigma
0.8	59.39ps	51.09ps	43.68ps	34.32ps	3.62E-23	3.76E-23	6.01ps	6.13ps
0.75	68.01ps	57.09ps	49.5ps	38.37ps	4.67E-23	4.72E-23	6.83ps	6.86ps
0.7	82.28ps	66.75ps	56.90ps	43.19ps	6.80E-23	6.00E-23	8.24ps	7.74ps
0.65	106.5	86.2	67.39ps	49.46ps	1.30E-22	9.22E-23	11.40ps	9.60ps

Execution times for the design of approximate adders with and without OpenMP are shown in Figures 6 and 7 respectively. A speed-up of about 1.61-fold using four threads is achieved on a quad-core processor, which is far less than linear. A speedup of at least two on four threads would have been promising. We believe this could be possible with the introduction of larger generation gap in *Delayed Elitism*.

Table 4.5: Results of Approximate Mirror Adder IV

Voltage	Sum Worst	Carry Worst	Sum Mean	Carry Mean	Sum Variance	Carry Variance	Sum Sigma	Carry Sigma
0.8	151.1ps	27.36ps	137.21ps	14.29ps	3.13E-23	4.59E-23	5.59ps	6.77ps
0.75	157.5ps	31.8ps	141.74ps	17.3287ps	3.95E-23	5.83E-23	6.29ps	7.64ps
0.7	167.2ps	37.29ps	147.38ps	21.15ps	5.71E-23	7.37E-23	7.56ps	8.59ps
0.65	184.8ps	42.85ps	155.41ps	25.81ps	1.15E-22	9.07E-23	10.7ps	9.54ps

When mutation was introduced in interconnections between LUTs, the developed circuit did not match the required configuration; the interconnections are important, as in a multiple bit adder, it is required for single bit adders to be in sequence so that the carry is propagated appropriately. The

power and performance numbers for individual approximate adder designs are listed in Table II. These numbers are obtained by HSPICE simulations.

Table 4.6: Propagation Delay and Power Consumption for Approximate Adders

Adder	Sum Worst(pS)	Carry Worst(pS)	Power (nW)
CMA	58.09	148	258.9419
AMA1	37.54	49.04	242.408
AMA2	56.41	49.15	122.37
AMA3	59.39	51.09	93.776
AMA4	58.78	52.12	96.882

From Figure 4.2, we can see that the difference between the GA run in a uniprocessor environment takes up to 700 seconds more than if it was run in a parallel environment. The speed up in such an environment is up to 1.6, this is the maximum we could achieve, [15] achieves a speedup of more than four for a 3:8 multiplexer, but this paper has been implemented using an adder in mind which consists of larger sub-elements and the order in which connections take place are important, hence parallelism cannot be implemented as frequently. Additionally, the results obtained by including OpenMP are shown in Figure 7. It can be observed that there is only a minimal increase in performance by using OpenMP.

The fitness of the evolved 4-bit Adder circuit is shown in Figure 8. It can be seen that the GA quickly converges to a fitness value of 303 out of a maximum fitness value of 424. In this case, the fitness evaluation takes up bulk of the execution time.

As approximate designs are targeted in this thesis, the adders are also evolved using a subset of the input space. In this case, the output is only calculated for 40% of the input space. Another attempt to restrict the design space of the GA was made by restricting the total number of gates to 16. The results for this case are shown in Figure 4.3. It can be observed that the highest fitness value is obtained in this case.

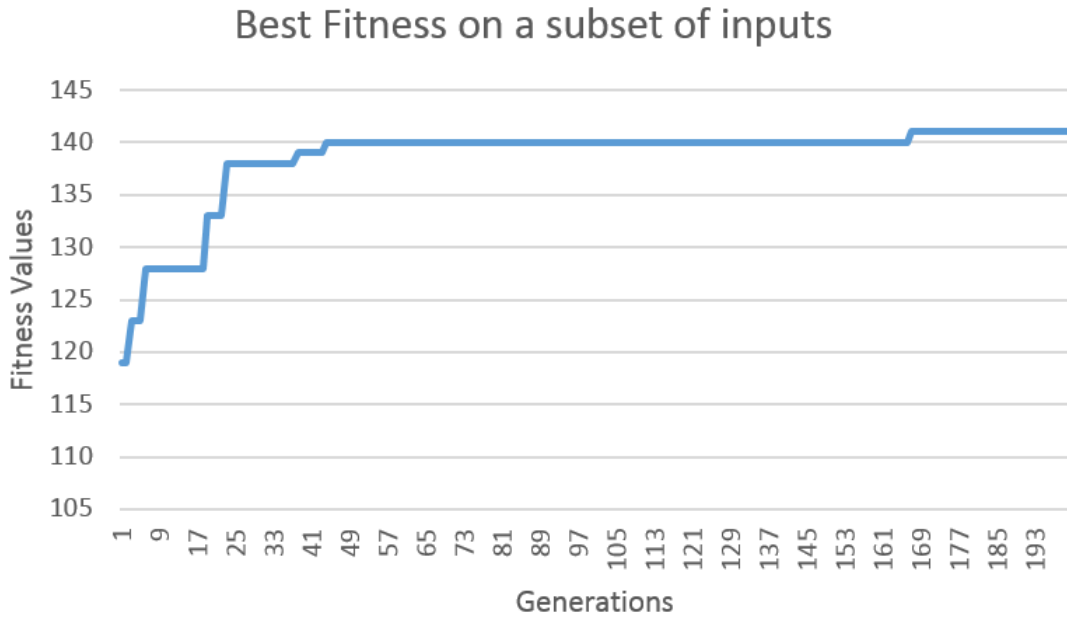


Figure 4.3: Fitness Evolved when on a subset of inputs restricted to maximum fitness value of 220

From Table 4.1 it can interpreted that the worst delays are observed in the critical paths of the adder, this is because the number of transistors in the critical path are the highest, which in turn increases the capacitance of the path which causes the larger delay also the 3 sigma value of the system is also very high. Table 4.2 shows better performance in terms of carry delay and even with 3 sigma values, the delay seems to be better than that of the accurate adder.

From Table 4.3 we can see that the performance of AMA-II is similar to that of AMA-I in terms of carry delay, but the sum delay is similar to that of the accurate adder, this is due to the fact that the sum path in the approximate adders analyzed here have more transistors than the carry path. Hence the carry is calculated faster than the sum. The data from Table 4.4 states that the Worst case delay of the sum over all the four voltages is worse than that of the accurate adder, but the worst case delay for carry is the best among the described adders.

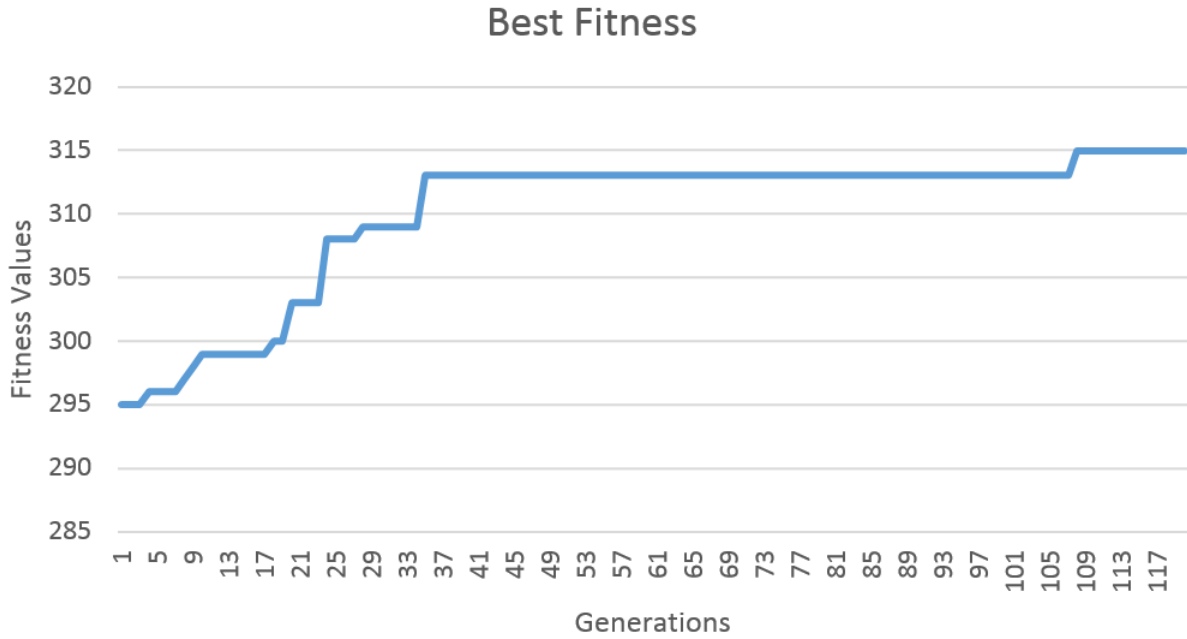


Figure 4.4: Fitness Evolved when the number of gates was restricted to 16 for a 4 bit adder

From Table 4.5 it can be inferred that AMA-IV has the best performance of the adders analyzed here in terms of carry delay performance, but the sum delay is very large, larger than even the accurate adder, this can again be explained due to the fact that the sum path is longer and has more transistors than the carry path, hence the delay is much larger.

Although the performance of the approximate adders surpasses that of the accurate adder even with process variation it should be noted that the approximate adders cannot replace accurate adders in all cases. They are only ideal in cases where there is no strict regimen on the accuracy of calculations, such as low quality video processing etc.

From the analysis of the results we can conclude that, process variation does not affect every adder equally. This gave us the idea of using logic diversity, to obtain the power/performance ratio, along with specified accuracy.

Table 4.7: Truth Table for Conventional CLA and Approximate CLA

Inputs			Accurate Outputs		Approximate Outputs	
A	B	C_{in}	Sum	C_{out}	Sum_1	C_{out1}
0	0	0	0	0	0	0
0	0	1	1	0	0	0
0	1	0	1	0	1	0
0	1	1	0	1	1	1
1	0	0	1	0	1	0
1	0	1	0	1	1	1
1	1	0	0	1	0	1
1	1	1	1	1	0	1

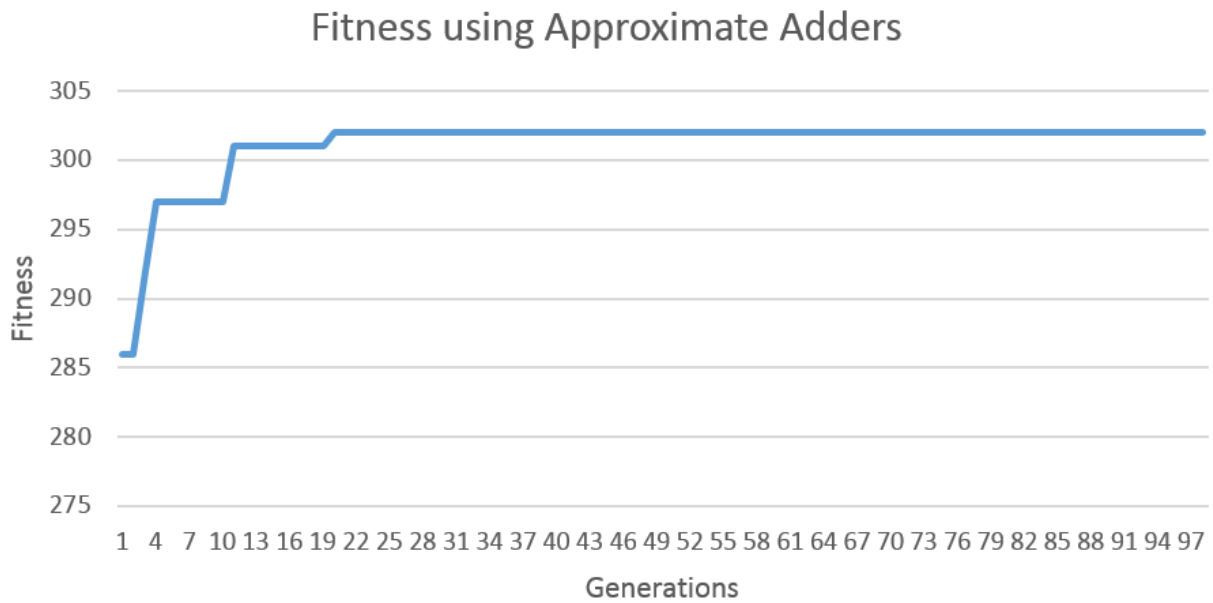


Figure 4.5: Fitness Evolved when Approximate Adders are used to implement a 4-bit Adder

This can be done using a GA which can be used to perform a search on the sample space and complete a number of iterations to determine the best candidates for each stage of the system.

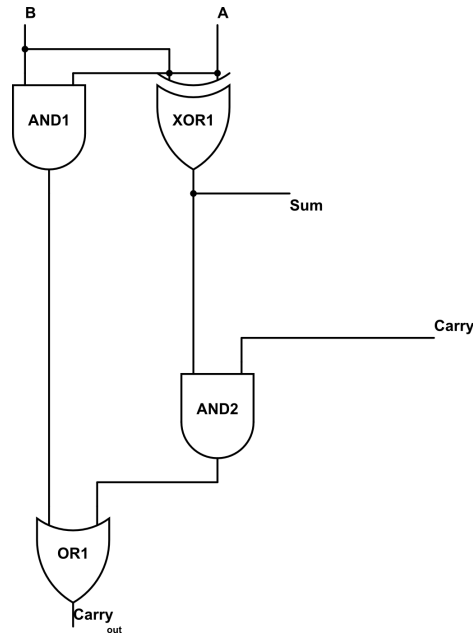


Figure 4.6: Approximate CLA Adder

CLA Adder

We created a new type of CLA Adder which produces approximate output, the results for the proposed CLA Adder are given in the table below. The CLA adder consists of all the elements usually associated with it.

We reduced the number of transistors in the sum part, where we directly tapped the input A as sum. This saves us around 16 transistors in total, with significant power and area savings. The Table 4.8 illustrates the changes in sigma value as the voltages are scaled to lower values. It shows that as the voltages decrease the variation in the expected delays increases drastically. For a decrease in voltage from 0.8V for 22nm prototype to 0.65V the sigma values increase from 7.51 to 18.4, which is almost two times that of the original value, making the circuit behavior even more unpredictable, on the other hand, this could be used to our advantage in approximate computing, by decreasing

the voltage and reducing the time required to latch on to the values could lead to interesting results.

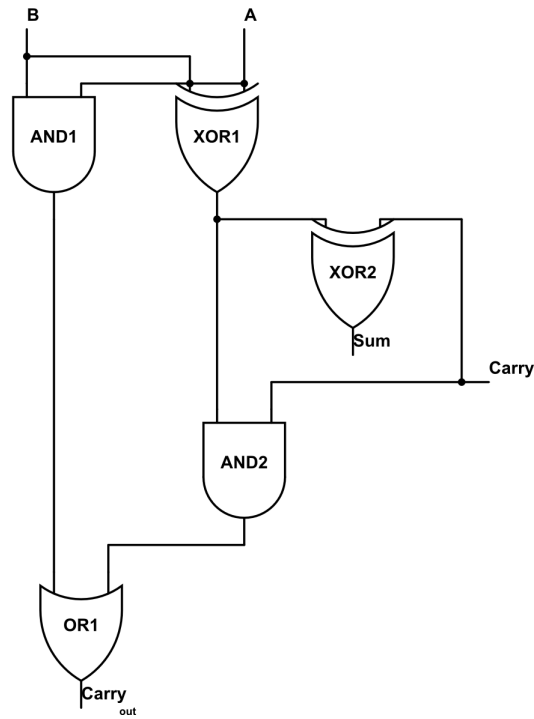


Figure 4.7: Accurate CLA Adder

Table 4.8: Results of Approximate CLA

Voltage	Sum Worst	Carry Worst	Sum Mean	Carry Mean	Sum Variance	Carry Variance	Sum Sigma	Carry Sigma
0.8	0	64.22ps	0	36.8215ps	0.00E+00	5.64E-23	0	7.51ps
0.75	0	81.59ps	0	47.0368ps	0.00E+00	8.50E-23	0	9.2181ps
0.7	0	119.3ps	0	61.5858ps	0.00E+00	1.49E-22	0	12.1989ps
0.65	0	199.5ps	0	83.8076ps	0.00E+00	3.39E-22	0	18.4029ps

There could be other areas to approximate in this type of adder, especially the carry propagation

part, which increases with complexity as the number of computation bits increase, but modifying the carry part could lead to changes in the carry output, as carry usually amounts to MSB, the output could be severely compromised even for small changes, which could not be satisfactory as the power saved would not be proportional to the accuracy lost.

CHAPTER 5: CONCLUSION

This thesis indicates benefit for parallelizing the GA and exploration of creative cascaded circuits such as adders where the current stage is heavily-dependent on the previous stage. Although we were able to achieve a modest speed-up of 1.61, in most cases, when the population size is increased then the effects of parallelization tends to be more pronounced.

Future work could include implementation of *DelayedElitism* which could improve the avenues where parallelism could be improved, in case of *DelayedElitism* the *parallel_invoke* method in TBB could be used to run fitness evaluation function and Evolution in parallel, this could improve the design-time significantly. Also, the GA could be modified to design larger circuits than the ones currently designed, these could then be implemented in chips where manual intervention for hardware faults is not possible, in such cases a GA could be remotely used to reconfigure the circuits when the systems aren't functioning appropriately.

The effect of process variation was also calculated. As the number of transistors is reduced, the distribution of the transistor widths and gate oxide may shift away from a Gaussian Curve. This result was demonstrated in different types of single-bit adders with the delay sigma increasing from 6psec to 12psec, and when the voltage is scaled to Near-Threshold-Voltage (NTV) levels the sigma increases by up to 5psec. Approximate Arithmetic Units were not affected more greatly by the change in distribution of the thickness of the gate oxide. Even when considering the 3-sigma value, the delay of an approximate adder remains below that of a precise adder with additional transistors. Additionally, it is demonstrated that the GA obtains innovative solutions to the appropriate combination of approximate arithmetic units, to achieve a good balance between energy savings and accuracy.

In the future, further enhancements could be implemented. Including testing out the 10nm FinFET as well as the 7nm FinFET which is currently on the road-map for Intel, as the sizes move towards the lower end the effect of process variation could be more pronounced in approximate circuits, but the current generation of transistors used in approximate adders have minimal effect on the performance of such adders.

Table 5.1: Summary of Approximate Adder Performance

Adder Type	Power	Variability	Test Inputs	NTV Performance
RCA (AMA-I)	Poor	High	Truth Table	Average
RCA (AMA-II)	Average	Low	Truth Table	Excellent
RCA (AMA-III)	Excellent	Average	Truth Table	Good
RCA (AMA-IV)	Excellent	Average	Truth Table	Average
CLA Adder	Average	Low	Truth Table	Poor

From Table 5.1, it can be inferred that when the approximate adders are carefully selected to be used in systems, the yields could be much higher than when only accurate circuits are used. It can also be noted that in certain cases the NTV performances of some of the approximate adders is much better than that of the accurate adder. In addition to approximate adders providing better power savings, their delays are also significantly lower than that of their accurate counterparts. An approximate ripple carry adder in certain could be faster than an accurate CLA Adder. As the variability of the approximate adders has also been proven to be not as high as to invalidate it in comparison to accurate adder, it is an added advantage to directly manufacture approximate adders. The worst case delay of an approximate adder with variability is still lower than that of the accurate adder, hence in cases where there is no strict need to obtain accurate results, the use of approximate adders could be promoted. In cases where the requirements in the output are variable and there is area redundancy then accuracy configurable units could be implemented, where the required accuracy of the output could be obtained on demand.

The comparison of OpenMP and Intel TBB is also done here. The results show that OpenMP is slightly faster than Intel TBB on eight threads, but it is not faster by a significant amount. This result could also be due to the workload of the CPU at any given time. Under certain circumstances when the OS is using up more resources, the allocation of work could become a bit skewed, this could result in different results when the number of threads is greater than the number of cores, but we repeated the experiment a number of times to obtain this result. It can also be confidently said that under normal circumstances when the APIs OpenMP and Intel TBB are used when the number of threads is equal to the number of cores available on the machine. This is due the fact that when the number of threads is greater than the number of cores the data is assigned in a random order over four cores, which causes slight overhead due to assignment of data in the pipeline. The methods of parallelizing either way is also different, the OpenMP concept uses barriers and pragmas to implement parallelism whereas Intel TBB uses a method which is also used in POSIX implementation i.e. it uses functions as a way of implementing parallelism. The functions are then appropriately multiplied and assigned to different threads automatically.

Future work could include implementation of a GA which could also perform HSPICE simulations to calculate its fitness value and use the results from the HSPICE simulation to determine the correct circuit to be implemented. This could be parallelized by running multiple instances of HSPICE. The main issue with such an implementation would be the ability of the simulation environment to create a netlist and to make sure that any netlist produced by such an environment follows the rules of HSPICE syntax with a high degree accuracy, the other challenge would be obtain the results from the output of HSPICE, this should be easier than generating the netlist, as the output usually contains the results after a certain keyword. Hence the environment could be programmed to perform a keyword search to find the required results. The third major issue with such a setup would be the time required to perform a fitness calculation, with a sample of 1000

Monte Carlo simulations, the simulation on HSPICE of any sizable circuit would be greater than 5 hours depending on the processing power of the machine used. If these challenges were to be solved a feasible GA could be made, which could make highly efficient circuits on demand.

APPENDIX : SIMULATION CODE


```

/*****
File - TestGen.cpp
*/

Function - Generates the input test vectors for a given multi bit adder
*

*****/
#include <iostream>
#include <vector>
#include <cstdlib>
#include <fstream>

int const    n = 16;
int const x = pow(2, n);
//const int NoofInputLines = 4;

using namespace std;
void calculateoutput(vector<vector< int > > &vInput, int &carry, vector< int > &bResult);

int main()
{
    ofstream TruthTable;
    ofstream Outputonly;
    ofstream bResultonly;
    using namespace std;
    //int const    n = 4;
    //int x = pow(2, n);
    int const    nBits = 1 << n;
    vector<vector< int >> InputVals;
    int powerOf2 = 1;
    vector <int> bResult;
    vector <int> bResult1;
    vector <int> bResult2;
    int carry = 1;
    TruthTable.open("./TruthTable_n-bit.txt");
    Outputonly.open("./Outputonly.txt");
    bResultonly.open("./BResult.txt");

```

```

int counter = 0;
int loopCounter = n / 2 - 1;

for (int i = 0; i < n; ++i)
{

    InputVals.push_back(vector < int >());
    for (int bitNum = 0; bitNum < nBits; ++bitNum)
    {

        //cout << 1 - bitNum / powerOf2 % 2;
        InputVals.at(i).push_back(1 - bitNum / powerOf2 % 2);
        cout << InputVals.at(i).at(bitNum);
        //cout << " ";

    }

    cout << endl;
    powerOf2 *= 2;

}

calculateoutput(InputVals, carry, bResult);
/*for (int i = x*2 - 2 ; i >= 0 ; i = i - 2)
{
    bResult2.push_back(bResult[i]);
}

for (int i = x*2 - 1; i >= 0; i = i - 2)
{
    bResult1.push_back(bResult[i]);
}*/
//InputVals.insert(InputVals.begin(),bResult);
//InputVals.insert(InputVals.begin(), bResult2);
//cout << bResult.size();

```

```

//InputVals.push_back(bResult1);
//InputVals.push_back(bResult2);
//
for (i = 0; i)

    for (int j = x - 1; j >= 0; j--)
    {
        for (int i = n - 1; i >= 0; i = i--)
        {

            //cout << InputVals.at(i).at(j) << " ";
            TruthTable << InputVals.at(i).at(j) << "␣";

        }
        //cout << InputVals.at(i).at(j + 1);
        for (int i = loopCounter; i >= (loopCounter - n/2 - 1) ; i--)
        {
            counter++;
            cout << counter << endl;
            cout << loopCounter << endl;
            if (counter >= n / 2 )
            {
                cout << "␣"<< bResult[i] << "␣";
                TruthTable << bResult[i] << "␣";
                Outputonly << bResult[i] << "␣";

                loopCounter += counter;
                counter = 0;
                break;
            }
            if (loopCounter > x*n)
                break;

            //cout << bResult[i] << " ";
            TruthTable << bResult[i] << "␣";
            Outputonly << bResult[i] << "␣";

        }
    }

```

```

        cout << endl;
        TruthTable << endl;
    }

    for (int i = 0; i < x * 2 - 1; i++)
        bResultonly << bResult[i] << " ";
    cout << bResult.size();
    TruthTable.close();
    Outputonly.close();
    bResultonly.close();
    system("pause");
}

/*****
File - SPFit.cpp

Function - Makes the TestGen output readable for the Main Program

*****/

# include<stdio.h>
# include<iostream>
#include <fstream>
#include <vector>
#include <string>

using namespace std;

void main()
{
    int inputs;
    ifstream input;
    ofstream output;
    char temp[256];
    string pushed[1000];
    input.open("./FPGA.txt");
    output.open("./Output.txt");

```

```

if (!input)
{
    cerr << "Couldn't Open the Settings File" << endl;
}

for (int j = 0; j < 120; j++)
{

    //for (int i = 0; i < 5; i++)
    {
        input >> inputs;
        input.getline(temp, 255);
        pushed[j] = temp;
        cout << pushed[j].size();
    }
}

cout << pushed[0][4];
for (int j = 0; j < 120; j++)
{
    for (int i = 1; i < 4; i++)
    {
        output << pushed[j][i];

    }
    output << endl;
}

output.close();
input.close();
system("pause");
}

```

```

/*****
File - DataStructures.h

Function - Provides Scaffolding for the main loop

*****/

#include <vector>
#include <string.h>
#include <fstream>
#include <iostream>
#include <math.h>
#include <time.h>

#include "tbb/parallel_for.h"
#include "tbb/blocked_range.h"
#include "tbb/concurrent_vector.h"
#include "tbb/spin_mutex.h"

#include <cstdlib>

using namespace std;
using namespace tbb;
using std::vector;

//Settings for FPGA and GA parameters
class ClsSettings
{
public:
    //FPGA settings
    string FilePath;//the settings input file path
    int NoOfCLBs;
    int NoOfLUTsPerCLB;
    int NoOfLUTSelectLines;
    int NoOfLUTOutputLines;
    int NoOfInputLines;
    int NoOfOutputLines;
    int nthreads; // number of threads

```

```

    int nruns; // number of runs

    //GA parameters settings
    int PopulationSize;
    int OffSpringSize;
    float MutationRate;
    float CrossoverRate;
    int NoOfGenerations;
    int TournamentSize;
    int ElitismSize;

    //Input and Output settings
    int NoOfConfigFiles;
    int NoOfGenerationOutput;

    //Read input file and fill into the parameters above
    bool ReadFile();
};

class ClsPortValue
{
public:
    vector <bool> vbTruthTableInput;
    vector <bool> vbTruthTableOutput;
};

/*****
//Defines the truth table inputs and outputs and reads their values from file
class ClsTruthTable
{
public:
    string FilePath; //the truth table's input file path
    string ModuleName;
    int NoOfInputPorts;
    int NoOfOutputPorts;
    int NoOfRows;
    vector <ClsPortValue> vcPortValue;

    //Read input file and fill values into Truth Table

```

```

        bool ReadFile();

        ~ClsTruthTable();
};

/*****
//Defines FPGA basic block unit (Look_Up_Table)
class ClsLUT
{
public:
    vector <bool> vInputValue;
    vector <int> vInputLine;
    vector <bool> vOutputValue;
    int FunctionType;
    bool CalculateOutput(vector <bool>,volatile int [], int carry);
    bool LutCopy(ClsLUT *, ClsLUT *, ClsSettings *);
};

/*****
//each Configurable_Logic_Block is given an ID
class ClsCLB
{
public:
    vector <ClsLUT *> LUTs;//each CLB is an array of LUTs
    //Functions
    bool DeleteCls();
};

/*****
//Each Individual has an ID and a chromosome
class ClsIndividual
{
public:
    vector <ClsCLB *> CLBS;
    vector <bool> vInput;
    vector <int> vOutputLine;
    vector <bool> vOutputValue;
    int iFitness;

    bool ConfigurationIndivi(ClsSettings * Object);

```



```

//Generate a Boolean function randomly
int AssignBooleanFunction();

//calculates each individual's fitness relative to the truth table output
bool CalculateFitness(ClsTruthTable * , ClsSettings *,bool );
bool ClsCleanAll();

//Does Copy operation for an individual
bool PerformCopy(ClsIndividual *,ClsSettings * );

//Output the Configuration of Individual
void ConfigOutput(ClsSettings * Object);

//Convert the function type to string
string FunctionType(int);

};

/*****
//defines a generation as a vector of individuals
class ClsGeneration
{
public:
    int ID;

    tbb::concurrent_vector<ClsIndividual *> Individuals;
    tbb::concurrent_vector<ClsIndividual *> vOffSpring;
    tbb::concurrent_vector<int> vOffSpringIndex;
    //tbb::concurrent_vector<int> vSelected;
    tbb::concurrent_vector<int> vIndividual;

    spin_mutex FitnessMutex;

    int MaxCalculatedFitness;
    int iMaxIndivi;

    bool InitializeGeneration(ClsSettings *);

    int Selection(ClsSettings *, ClsGeneration * );

    bool PerformCrossover(ClsIndividual *,ClsIndividual *,ClsIndividual *,ClsSettings *);

```

```

    bool PerformMutation(ClsIndividual *,ClsSettings *);

    bool PerformElitism(ClsIndividual *,ClsSettings *);

private:

};

/*****
File - Coutput.cpp

Function - Calculates the Output and compares the output with the input

*****/

#include "DataStructures.h"

/*****AA(Note to Self) - Do Not Modify!! Checked !*****/
//According to the inputs and lut functionality
//get the LUT output
bool ClsLUT::CalculateOutput (vector <bool> vInput,volatile int bResult[], int carry)
{
    bool bReturn=false;
    //int iIndex=0;
    volatile int caser;
    vInput[2] = carry;
    if ((vInput[0] == 0 && vInput[1] == 0 && vInput[2] == 0))
        caser = 0;
    else if ((vInput[0] == 0 && vInput[1] == 0 && vInput[2] == 1))
        caser = 1;
    else if ((vInput[0] == 0 && vInput[1] == 1 && vInput[2] == 0))
        caser = 2;
    else if (vInput[0] == 0 && vInput[1] == 1 && vInput[2] == 1)
        caser = 3;
    else if (vInput[0] == 1 && vInput[1] == 0 && vInput[2] == 0)
        caser = 4;
    else if (vInput[0] == 1 && vInput[1] == 0 && vInput[2] == 1)

```

```

        caser = 5;
else if (vInput[0] == 1 && vInput[1] == 1 && vInput[2] == 0)
    caser = 6;
else if (vInput[0] == 1 && vInput[1] == 1 && vInput[2] == 1)
    caser = 7;
switch (FunctionType)
{

    //AMA 1
    case 0:
        if (vInput.size() < 3)
        {
            cout<< "InputSize_error";
            return bReturn;
            exit(0);
        }
        // begin truth table
        else
        {
            if (caser == 0 || caser == 4)
            {
                bResult[0] = 0;
                bResult[1] = 0;
            }
            else if (caser == 1)
            {
                bResult[0] = 1;
                bResult[1] = 0;
            }
            else if (caser == 2 || caser == 3 || caser == 5 || caser == 6)
            {
                bResult[0] = 0;
                bResult[1] = 1;
            }

            else if (caser == 7)
            {
                bResult[0] = 1;
                bResult[1] = 1;
            }
        }
    }
}

```

```

        }

    }

    // end truth table

    break;

// Approximate Adders 2
case 1:
    if (vInput.size() < 3)
    {
        cout<< "InputSize_error";
        return bReturn;
        exit(0);
    }

    else
    {
        if (caser == 3 || caser == 5 || caser == 6 || caser == 7)
        {
            bResult[0] = 0;
            bResult[1] = 1;
        }

        else if (caser == 0 || caser == 1 || caser == 2 || caser == 4)
        {
            bResult[0] = 1;
            bResult[1] = 0;
        }

    }

    bReturn=true;
    break;

// Approx Adder 3
case 2:
    if (vInput.size() < 3)
    {
        cout<< "InputSize_error";
        return bReturn;
        exit(0);
    }

```

```

else
{
    if (caser == 0 || caser == 1 || caser == 4)
    {
        bResult[0] = 1;
        bResult[1] = 0;
    }

    else if (caser == 2 || caser == 3 || caser == 5 || caser == 6 || caser == 7)
    {
        bResult[0] = 0;
        bResult[1] = 1;
    }
}

bReturn=true;
break;
// Approx Adder 4
case 3:
    if (vInput.size() < 3)
    {
        cout<< "InputSize_error";
        return bReturn;
        exit(0);
    }

    else
    {
        if (caser == 0 || caser == 2)
        {
            bResult[0] = 0;
            bResult[1] = 0;
        }

        else if (caser == 1 || caser == 3)
        {
            bResult[0] = 1;
            bResult[1] = 0;
        }

        else if (caser == 4 || caser == 5 || caser == 6)

```

```

        {
            bResult[0] = 0;
            bResult[1] = 1;
        }

        else if (caser == 7)
        {
            bResult[0] = 1;
            bResult[1] = 1;
        }
    }
    bReturn=true;
    break;

default:
    cout << "Error_in_the_LUT_calculation_function"<<endl;
    bReturn=false;
    break;
}

return bReturn;
}

//Copy current LUT to secondLut LUT
bool ClsLUT::LutCopy (ClsLUT * FirstLut, ClsLUT * SecondLut, ClsSettings * Settings)
{
    bool bResult=true;
    int iIndex=0;

    SecondLut->FunctionType = FirstLut->FunctionType;

    for (iIndex=0; iIndex < Settings->NoOfLUTSelectLines ; iIndex++)
    {
        SecondLut->vInputLine[iIndex]= FirstLut->vInputLine[iIndex];
    }

    return bResult;
}

```

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